**Instruction sheet for Part I of the 2007 High School Math Contest**

1. Your answer sheet will be machine graded. Please follow carefully the instructions printed on the answer sheet.

2. Make sure that your digit code number has been correctly recorded. Fill in the circles completely and darkly.

3. Your score on Part I of the Math Contest will be the number of correct answers.

4. Scientific calculators are permitted on Part I, but portable computers and pocket organizers are not.

5. Proctors are not permitted to discuss exam questions.

6. You have 80 minutes to complete Part I. When the proctor requests you to stop, you must stop working immediately and turn in your answer sheet.

7. You may begin the exam as soon as you have completed reading these instructions.

**High School Mathematics Contest**

**Part I**

**March 7, 2007**

1. $e^{-2\ln x}$ is equal to

   **A**: $-2x$

   **B**: $\dfrac{1}{x^2}$

   **C**: $-x^2$

   **D**: $\dfrac{-2}{x}$

   **E**: $-2\ln x$

2. If $0 < x < \dfrac{\pi}{2}$ then $\dfrac{\sin x}{1+\cos x}$ is equal to

   **A**: $\sin x + \tan x$

   **B**: $\cos\dfrac{x}{2}$

   **C**: $1 + \sin^2 x$

   **D**: $\dfrac{1-\cos x}{\sin x}$

   **E**: $\dfrac{1+\sin x}{1-\cos x}$

3. If $f(x) = \dfrac{x+1}{x-1}$ and if $g$ is the inverse function of $f$, then $g(x) =$

   **A**: $\dfrac{x}{x-1} - \dfrac{1}{x-1}$

   **B**: $\dfrac{x-1}{x+1}$

   **C**: $\dfrac{1+x}{x-1}$

   **D**: $\dfrac{1-x}{1+x}$

   **E**: $\dfrac{x}{2x+1}$

4. If $\ln x = 2$ then

   **A**: $x = e^2$

   **B**: $\ln 2 = x$

   **C**: $\ln 2x = x + \ln 2$

   **D**: $e^x = 2$

   **E**: $e = x^2$

5. If the radius $r$ of a circle is doubled, then the area is

   **A**: Multiplied by 2

   **B**: Multiplied by $\pi$

   **C**: $\pi r^2$ square units larger

   **D**: Multiplied by $2\pi r$

   **E**: Multiplied by 4

6. A certain population is growing by 6% per year. If $N$ is the population today, then the population 4 years from now is given by

   **A**: $N \times 0.06 \times 4$

   **B**: $N \times 1.06^4$

   **C**: $N + 0.24N$

   **D**: $(N + 0.06)^4$

   **E**: $N \times 6^4$

7. Which of the following statements are true?

   **A**: $\sin x$ is increasing for $0 \leq x \leq \dfrac{\pi}{2}$

   **B**: $\sin x$ is increasing for $0 < x < \dfrac{\pi}{4}$

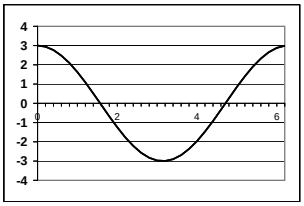
   **C**: $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

   **D**: All of the above

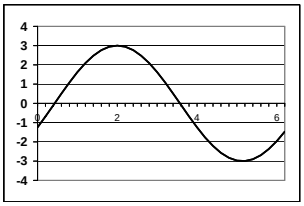
   **E**: None of the above

8. Which of the following is the graph of $3\cos(x-2)+2$ for $0 \leq x \leq 2\pi$?

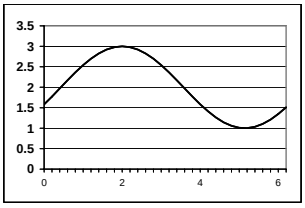
**A**:



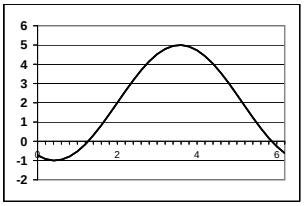

**B**:



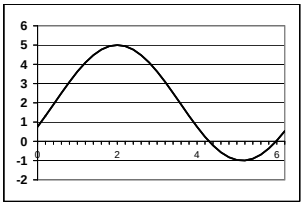

**C**:




**D**:




**E**:



9. If $x$ is positive, which of the following is largest?

   **A**: $\dfrac{x}{x+1}$

   **B**: $\dfrac{x}{x^2+1}$

   **C**: $\dfrac{x+2}{x+1}$

   **D**: $\dfrac{x^2+x}{x^2+x+2}$

   **E**: $\dfrac{1}{2}$

10. $f$ is a function whose graph is a straight line with slope 3. If $f(2) = -1$, then $f(4) =$

    **A**: 12

    **B**: 11

    **C**: 7

    **D**: 6

    **E**: 5

11. The planet Xenon, like Earth, has a spherical shape. The radius of the Earth is 4000 miles. The radius of Xenon is 4000 miles and 1 foot. The length of the equator of Xenon is

    **A**: Just over 6 feet longer than the equator of Earth.

    **B**: Almost double the length of the equator of Earth.

    **C**: 5281 feet longer than the equator of Earth.

    **D**: Just over 36 feet longer than the equator of Earth

    **E**: Within one foot of the length of the equator of Earth.

12. The complex number $i$ has the property that $i^2 = -1$. Then $i^{207} =$

    **A**: 1

    **B**: $-1$

    **C**: 0

    **D**: $i$

    **E**: $-i$

13. $\left(\sqrt{13}-\sqrt{12}\right)+\left(\sqrt{14}-\sqrt{13}\right)+\left(\sqrt{15}-\sqrt{14}\right)+\cdots+\left(\sqrt{209}-\sqrt{208}\right)=$

**A**: $\sqrt{209}$
**B**: $\dfrac{\sqrt{208}}{\sqrt{13}}$
**C**: $17$
**D**: $\dfrac{1-\sqrt{12}^{209}}{1-\sqrt{12}}$
**E**: $\dfrac{197}{\sqrt{209}+\sqrt{12}}$

14. Which of the following is a factor of $x^2-2x+2$?

**A**: $x-1$
**B**: $x+1-\sqrt{3}$
**C**: $x-\dfrac{\sqrt{3}}{2}$
**D**: $x-1-\sqrt{3}$
**E**: None of the above.

15. $\ln\dfrac{2^x}{x^2}$ simplifies to

**A**: $\dfrac{e^x}{x^2+2}$
**B**: $\ln 2^x+\ln x^2$
**C**: $x\ln 2-2\ln x$
**D**: $\dfrac{x\ln 2}{2\ln x}$
**E**: $\left(\ln\dfrac{2}{x}\right)^{x/2}$

16. In order to complete the square for $x^2-\dfrac{x}{2}$ we need to add

**A**: $\dfrac{1}{4}$
**B**: $-\dfrac{1}{4}$
**C**: $\dfrac{1}{2}$
**D**: $\dfrac{1}{16}$
**E**: $\dfrac{1}{8}$

17. A rectangle of length $L$ and width $W$ sits inside a larger rectangle of triple the length and triple the width. What is the area of the region between the two rectangles?

    **A**: $9LW$

    **B**: $2LW$

    **C**: $3L + 3W$

    **D**: $L^2W^2$

    **E**: $8LW$

18. Recall that $\cos 2x = \cos^2 x - \sin^2 x$. Then $\sin^2 x =$

    **A**: $\frac{1}{2}(1 - \cos 2x)$

    **B**: $\frac{1 + \sin 2x}{2}$

    **C**: $\frac{1}{2}(1 + \sin 2x)$

    **D**: $2 \sin x \cos x$

    **E**: $\frac{\cos x - \sin x}{2(\cos x + \sin x)}$

19. The big triangle in the figure below is isosceles, and the angle at the top has degree measure 30. The big triangle and the smaller triangle are similar. Find the degree measure of the angle marked $\alpha$.

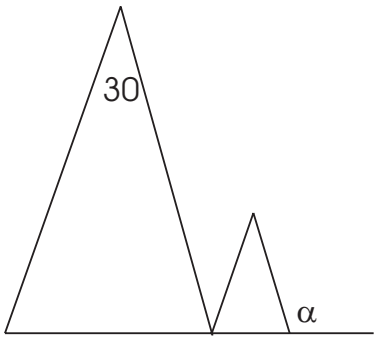
    **A**: 30

    **B**: 105

    **C**: 150

    **D**: 100

    **E**: 115



20. How many 4-letter "words" can be made using the letters bill? (Any sequence of these four letters counts as a "word". libl is one example.)

    **A**: 6

    **B**: 10

    **C**: 8

    **D**: 12

    **E**: 24

21. The solutions of $\sqrt{3}\sin x = 2\sin x \cos x$ for $0 \leq x \leq \pi$ are

    **A**: $x = \dfrac{\pi}{6}$

    **B**: $x = 0$, $x = \pi$, and $x = \dfrac{\pi}{6}$

    **C**: $x = 0$, $x = \dfrac{\pi}{6}$, and $x = \dfrac{\pi}{3}$

    **D**: $x = 0$, $x = \dfrac{\pi}{3}$, and $x = \pi$

    **E**: $x = 0$, $x = \pi$, and $x = \dfrac{\pi}{2}$

22. $3^3 + 3^4 + \cdots + 3^{100} =$

    **A**: $\dfrac{3^{101} - 1}{2}$

    **B**: $\dfrac{27}{2}(3^{98} - 1)$

    **C**: $\dfrac{1}{1 - \frac{1}{3}}$

    **D**: $\dfrac{3^{100} - 3^3}{2}$

    **E**: $9\dfrac{3^{101} - 1}{2}$

23. Exactly one of the following numbers is prime. Which is it?

    **A**: $107^4 - 1$

    **B**: $6^{33} + 3^{44}$

    **C**: $2^{61} - 1$

    **D**: $4^{73} - 1$

    **E**: $17^{63} - 7^{31}$

24. What is the smallest positive integer $n$ such that $3^n - 1$ is divisible by 7?

**A**: $n = 5$

**B**: $n = 12$

**C**: $n = 9$

**D**: $n = 7$

**E**: $n = 6$

25. The solution of $4^x - 2^{x+1} - 15 = 0$ is:

**A**: $x = \dfrac{1 \pm \sqrt{61}}{2}$

**B**: $x = \ln \dfrac{1 \pm \sqrt{61}}{2}$

**C**: $x = \dfrac{\ln 5}{\ln 2}$

**D**: $x = \dfrac{1 + \ln \sqrt{61}}{\ln 2}$

**E**: $x = \ln \dfrac{5}{2}$ and $x = -\ln \dfrac{5}{2}$