Master's Exam
General Topology
January 1993

1. Let X and Y be topological spaces and $f : X \to Y$ a function. Prove that f is continuous if and only if for every subset A of X, one has $f(\bar{A}) \subseteq \overline{f(A)}$.

2. A space X is locally path connected if for every point $x \in X$ and every open neighborhood U of x there is a path conencted open neighborhood V of x such that $V \subseteq U$. Prove that every conencted, locally path connected space is path connected.

3. Prove that every compact Hausdorff space is normal.

4. Prove that every separable metric space satisfies the second axiom of countability.

5. Give the set R of real numbers the standard topology. Prove that a subset A of R is compact if and only if it is closed and bounded.