How to Read Mathematics for Meaning –
Making Sense of Mathematical Prose

This part of the RaMMP materials reflects a compilation of the ideas that have been written about widely on the subject of reading mathematics. I doubt we could give credit to any one single person for a developing a "method" of how to read mathematics for meaning. So, we claim nothing really new here – but hopefully the manner in which it is presented and discussed will be useful to you as an entry-level mathematics student. Several links are also given to elaborate on the points being made. Obviously, you can't read them all; try a few! The final portion is a collection of what we hope are "Useful Tips" for reading mathematics.

Perhaps the most fundamental issue involved in reading mathematics for meaning for us is "making sense" of the mathematical words, phrases, and applications. What does it mean to "make sense" of the mathematical jargon used in the textbook? When we start to try to answer questions like this one, it becomes clearer that the vocabulary, sentences, and uses of mathematical terms are different than other academic disciplines. Therefore, we should not attempt to read mathematics as we would other types of reading. Many times students don't understand this difference and approach reading mathematics as they would read literature, for example. The point here is that reading mathematics for meaning requires special unique skills that are different from reading other matter; you must learn to read mathematically. The phrase describing this is *reading protocol*, a collection of strategies that the reader must use in order to make sense of the words and phrases. Reading mathematically is more than reading the printed words on the page. It requires linking these words with the mathematical ideas that are involved. So, making sense of mathematical prose is a complex process involving understanding mathematical terms *and* the ideas connecting them.

A very informative and well-written background paper "Reading and Writing in the Mathematics Classroom" by Mark Freitag is found at the URL

http://math.coe.uga.edu/TME/Issues/v08n1/3freitag.pdf

An important point that he makes is that mathematical reading is not linear, read from cover to cover; rather, mathematical reading may require the reader to reread passages many times in order to make sense of them. "Students should realize that time and effort are needed to completely comprehend mathematical text, and so they need to exercise patience, concentration, and determination" (p. 17). Freitag also offers some useful comments related to the connections between reading and writing mathematics. Another interesting background paper "How to Read Mathematics" by Shai Simonson and Fernando Gouvea can be found at the URL

http://www.stonehill.edu/compsci/History_Math/math-read.htm

One of Simonson's and Gouvea's initial points is "Don't Miss the Big Picture" which means "Don't assume that understanding each phrase, will enable you to understand the whole idea. … A math article has a story! Try to see what the story is before you delve

into the details. You can go in for a closer look once you have the framework to fill with details." (p.1-2).

Useful Tips for Reading Mathematics

Now let's concentrate on some specific suggestions related to reading mathematics, whether it be the textbook in general or specific problems, exercises, or applications, e.g., word problems. We've seen from the above statements that reading mathematics is different than reading other material and so we have to develop different strategies for reading it. The good news is that if you will make a conscientious effort to develop some new strategies for reading mathematics, the chances are you will be successful! Believe it or not, but some of your everyday reading habits are "bad habits" when it comes to reading mathematics.

Make yourself a list of these suggestions and try to apply them when you are reading your mathematics.

- **Focus your attention on the "Big Picture", the concepts in the section.**
  Many students approach mathematics by going to the assigned exercises *first*, leaving the prerequisite reading in the section to be done later on (perhaps) and then, only if needed, to work the exercises. This may seem to be a time-efficient method to complete assigned homework, but it is a bad behavior that needs to be addressed.
  Don't be frustrated if you are confused during your initial reading of the section. Remember, you are looking for the "Big Picture" in the section; try to identify these major concepts that are being discussed.

- **Reread the section.**
  So here is a major difference between reading mathematics and other materials, e.g., a novel. You must go back and reread the section - perhaps several times, in fact - to build on your understanding of the "Big Picture" and fill in some of the details. If you do not understand a particular sentence, figure out why. Is there a specific word or term that you do not understand? You may have to refer to an earlier section in the textbook or a glossary of terms to help you fill in this detail.

- **Write notes to yourself**.
  Reading (and rereading) requires action; you are doing something as part of the reading process. You should plan to take notes in an organized fashion; these are *your* notes, the vehicle that helps you connect your understanding with the textbook material. Your notes are personal, meaningful to you; they *include in your own words* what you understand, don't understand, and questions that you need to resolve. They also allow you an opportunity to fill in any details that you found missing in the reading material. Remember that mathematical language is very precise; words used a mathematical context

often do not have the same meaning as in everyday use. Your notes can help you make sense of mathematical vocabulary. This process of "communicating with yourself" in writing is the cornerstone of *making sense* of mathematics. Yes, it is not easy – and yes – it takes time!

Only after you feel you have an understanding of the section concepts and how they fit together, should you attempt to work the exercises and problems. And, yes, if needed, you should use the notes you have written to yourself to complete your homework.

- **Study the examples and figures.**
  Most textbooks include examples with detailed solutions; many also include sample problems following these worked problems. Pay attention to how the figures relate to the "Big Picture" what aspects of the figures are important in understanding the key concepts. Examples and figures provide an excellent opportunity for you to assess your readiness to begin the assigned exercises at the end of the section.

- **Approach applications with a positive attitude**.
  Most applications are in the form of word problems – and who likes them! Let's remember what we've done so far to get ready to work the problems. We have read, and reread, the textbook section probably several times; we have the "Big Picture" in mind and a collection of notes that helps us connect the concepts and understand the language used; and we have studied examples of the concepts. In essence, we have a roadmap for how to approach problems involving the concepts discussed in this section.

  So, what is holding us back? Most likely, the roadblock is *reading* – understanding the statement of the problem and what it is asking us to do. We can overcome this problem by employing the strategies we have discussed above. Namely, read the problem initially for the "Big Picture"; reread it with the intent of filling in some of the details; write yourself some notes about the problem, e.g., what is being asked for? What vocabulary needs to be clarified? Many applied problems are very vocabulary-specific; you must know very precise definitions of terms before you can begin. You will discover this during your initial reading of the problem and can consult other sources (index, appendix, dictionary) to help you write your notes.

  If you have systematically employed these reading strategies for the section and the statement of the application, your chances of success are greatly enhanced.

- **Ask for help as a last resort.**
  Even in the best of times, we sometimes do not meet with success. Being successful as a result of our own individual efforts is very rewarding. If, after

all of your efforts, you still are not successful, bring your notes and questions to your instructor for guidance.

For further elaboration on these suggestions, please refer to the following URLs

http://www.math.uh.edu/~tomforde/MathReadingTips.pdf

http://www.scs.tamu.edu/selfhelp/elibrary/math_study_skills.asp

http://web.centre.edu/alexmcal/docs/reading.html

Finally, this discussion has focused on reading mathematics as a fundamental step in the process of solving problems. A classic reference to problem solving is How to Solve It by G. Polya (Princeton University Press, Princeton, NJ, 1973). The URL below summarizes Polya's method for solving problems.

http://www.math.utah.edu/~pa/math/polya.html

Good Luck!

Credits. *The Reading and Modeling Mathematical Problems (RaMMP) project* is made possible through funding to Oklahoma State University from the Howard Hughes Medical Institute. It is based on work sponsored wholly, or in part, by the Howard Hughes Program for Retention of Undergraduates in the Biological Sciences at Oklahoma State University.
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