

CALCULUS-II, MATH 2153-006, 22-JAN-2009

QUIZ-1

Evaluate the integral

$$\int \frac{\ln y}{\sqrt{y}} dy.$$

1st Solution :- Use integration by parts :-

Let $u = \ln(y)$ then $du = \frac{1}{y} dy$

& $dv = \frac{1}{\sqrt{y}} dy$ $v = 2\sqrt{y}.$

$$\begin{aligned} \therefore \int \frac{\ln y}{\sqrt{y}} dy &= \int u dv = uv - \int v du = 2\sqrt{y} \ln y - \int 2\sqrt{y} \cdot \frac{1}{y} dy \\ &= 2\sqrt{y} \ln y - 2 \int \frac{1}{\sqrt{y}} dy = \boxed{2\sqrt{y} \ln y - 4\sqrt{y} + C} \end{aligned}$$

2nd Solution :- Put $y = t^2$ then $dy = 2t dt$, $\ln y = \ln t^2 = 2 \ln t$

$$\begin{aligned} \therefore \int \frac{\ln y}{\sqrt{y}} dy &= \int \frac{2 \ln t}{t} \cdot 2t dt = 4 \int \ln t dt \\ &= 4 (t \ln t - t) + C \\ &= 4\sqrt{y} \cdot \ln \sqrt{y} - 4\sqrt{y} + C \end{aligned}$$

$$\ln \sqrt{y} = \ln(y^{1/2}) = \frac{1}{2} \ln y$$

$$= \boxed{2\sqrt{y} \ln y - 4\sqrt{y} + C.}$$

Note:- In the integrand we see "ln y" which implies that y is "positive". This gives us the freedom to express the final answer in terms of \sqrt{y} and $\ln y$.
Otherwise, technically, we would have to write $\ln|y|$...etc.