

CALCULUS-II, MATH 2153-006, 5-FEB-2009  
QUIZ-2

Evaluate the integral

$$\int \frac{1}{e^x + 1} dx.$$

Put  $u = e^x + 1$ . Then  $du = e^x dx \Rightarrow dx = \frac{du}{e^x} = \frac{du}{u-1}$ .

$$\therefore \int \frac{dx}{e^x + 1} = \int \frac{du}{u(u-1)}$$

Now integrate ~~by parts~~ using partial fractions:-  $\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$

$$\Rightarrow 1 = A(u-1) + B \cdot u$$

Put  $u=1$  to get  $B=1$

Put  $u=0$  " "  $A=-1$

$$\therefore \frac{1}{u(u-1)} = -\frac{1}{u} + \frac{1}{u-1} = \frac{1}{u-1} - \frac{1}{u}$$

$$\int \frac{du}{u(u-1)} = \int \frac{du}{u-1} - \int \frac{du}{u} = \ln|u-1| - \ln|u| + c$$

$$\int \frac{dx}{e^x + 1} = \ln|e^x + 1 - 1| - \ln|e^x + 1| + c = \ln|e^x| - \ln|e^x + 1| + c.$$

$$= \boxed{x - \ln(e^x + 1) + c.}$$

Note:-

$$e^x > 0 \text{ for all } x$$

$$\Rightarrow \ln|e^x| = \ln(e^x) = x.$$

$$\ln|e^x + 1| = \ln(e^x + 1)$$