SOLUTIONS

CALCULUS-II, MATH 2153-006, 26-FEB-2009
QUIZ-4

Determine whether the series converges or diverges:

(1)

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}} \quad - \quad \text{Convergent}$$

(2)

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n} \quad - \quad \text{Convergent}$$

(1) Compare $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

p-series with $p = \frac{3}{2} > 1 \Rightarrow$ convergent.

comparison is justified via the limit comparison test:

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^3+1}} \Big/ \frac{1}{\sqrt{n^3}} = \lim_{n\to\infty} \frac{\sqrt{n^3}}{\sqrt{n^3+1}} = \lim_{n\to\infty} \sqrt{\frac{1}{1+1/n^3}} = 1.$$

(2) ~~Apply~~ the Alternating series test to $\sum_{n=2}^{\infty} (-1)^{n-1} \frac{\ln(n)}{n}$.

$b_n = \frac{\ln(n)}{n}$.

- $b_3 > b_3 > \cdots > b_n > \cdots$, $f(x) = \frac{\ln(x)}{x} \Rightarrow f'(x) = \frac{x \cdot \frac{1}{x} - \ln(x)}{x^2} = \frac{1-\ln(x)}{x^2} < 0$

$\Rightarrow f(x)$ is decreasing

$\Rightarrow b_n$ is decreasing.

- $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{1/n}{1} = 0.$

$\therefore$ Alternating series test applies $\Rightarrow$ Series converges.