

TOPICS IN GEOMETRY: SHEAF THEORY
MATH 6490, SPRING 2009
HOMEWORK 2

Exercise 1. Let F be a field, and (V_\bullet, d_\bullet) be a complex of finite-dimensional F -vector spaces. Assume that it is a finite complex, i.e., $V_n \neq 0$ for only finitely many n . Show that

$$\sum_i (-1)^i \dim_F(V_i) = \sum_i (-1)^i \dim_F(H_i(V)).$$

Exercise 2. State and prove the *snake lemma*.

Exercise 3. State and prove the *five lemma*.

Exercise 4. Consider two short exact sequences

$$S: 0 \longrightarrow N \longrightarrow A \longrightarrow M \longrightarrow 0$$

and

$$T: 0 \longrightarrow M \longrightarrow B \longrightarrow K \longrightarrow 0.$$

Note that T starts where S ends. Describe how we might *splice* these two sequences to get an exact sequence

$$S \circ T: 0 \longrightarrow N \longrightarrow A \longrightarrow B \longrightarrow K \longrightarrow 0$$

This operation is called the Yoneda composition of two short exact sequences to get a longer sequence. Philosophically, this is the *same* as wedging two differential one-forms to get a differential two-form. Which is why this composition is sometimes called the wedge product of two short exact sequences. (For more on Yoneda composition, see MacLane's classic 'Homology'.)
