TOPICS IN GEOMETRY: SHEAF THEORY
MATH 6490, SPRING 2009
HOMEWORK 3

For this homework set F will denote an additive functor from R-**MOD** to **AB**.

Suppose F is covariant; we say F is *left exact* if for any short exact sequence

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

of R-modules, the sequence $0 \longrightarrow F(A) \longrightarrow F(B) \longrightarrow F(C)$ is exact. In the case when F is contravariant, we would ask for $0 \longrightarrow F(C) \longrightarrow F(B) \longrightarrow F(A)$ to be exact. (You can remember this by noting that after applying F, the 0 is only on the left.) Right exactness is defined similarly: When F is covariant, we say F is *right exact* if for any short exact sequence $0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$ of R-modules, the sequence $F(A) \longrightarrow F(B) \longrightarrow F(C) \longrightarrow 0$ is exact. In the case when F is contravariant, we would ask for $F(C) \longrightarrow F(B) \longrightarrow F(A) \longrightarrow 0$ to be exact. (You can remember right exactness by noting that after applying F, the 0 is only on the right.)

Exercise 1. If F is left exact then show that R^0F is naturally equivalent to F. (You have to separately argue the two cases depending on when F is covariant or contravariant.)

Exercise 2. If F is right exact then show that L_0F is naturally equivalent to F. (You have to separately argue the two cases depending on when F is covariant or contravariant.)

Exercise 3. Let M be a projective module, and F an additive covariant functor. Then show that

$$L_nF(M) \simeq \begin{cases} F(M) & \text{, if } n = 0 \\ 0 & \text{, if } n \geq 1. \end{cases}$$

Exercise 4. Let I be an injective module, and F an additive covariant functor. Then show that

$$R^nF(I) \simeq \begin{cases} F(I) & \text{, if } n = 0 \\ 0 & \text{, if } n \geq 1. \end{cases}$$

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