TOPICS IN GEOMETRY: SHEAF THEORY
MATH 6490, SPRING 2009
HOMEWORK 6

Exercise 1. Let M be a Riemann surface and $\mathcal{O}_M$ the structure sheaf on M. Show that for all points $x \in M$ the stalk $\mathcal{O}_{M,x}$ is isomorphic to the subring of the ring $\mathbb{C}[[Z]]$ consisting of all those power series in the variable Z which have a positive radius of convergence.

Exercise 2. Let Ω be a lattice in $\mathbb{C}$ and consider the canonical projection $\pi : \mathbb{C} \to \mathbb{C}/\Omega$. Show that π is a morphism of complex manifolds. Show that π induces an isomorphism at the level of stalks of the corresponding structure sheaves. Does this mean that π is an isomorphism? Why or why not?

Exercise 3. Let μ_n be the group of n-th roots of unity generated by $\zeta = e^{2\pi i/n}$. Consider the action of ζ (and hence of μ_n) on $\mathbb{C}$ by $z \mapsto \zeta z$. Show how you can make $\mathbb{C}/\mu_n$ into a manifold. (See Example 17 on p.42 of Harder's book.)

Exercise 4. *(Skyscraper sheaves.)* Let X be a topological space, P be a point in X, and A be an abelian group. Define the skyscraper sheaf $\mathcal{S}_P$ on X determined by the point P and the group A as: $\mathcal{S}_P(U) = A$ if $P \in U$ and $\mathcal{S}_P(U) = 0$ if $P \notin U$, for any open set U. Determine the stalks of $\mathcal{S}_P$. (Note: It is not assumed that X is Hausdorff, or for that matter X need not even be a T_1-space.)

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