

**TOPICS IN GEOMETRY: SHEAF THEORY**  
**MATH 6490, SPRING 2009**  
**HOMEWORK 9**

**Exercise 1.** Let  $X$  be the disjoint union of the topological spaces  $X_1$  and  $X_2$ . For any abelian group  $A$ , show that

$$H^q(X, A) \simeq H^q(X_1, A) \oplus H^q(X_2, A).$$

**Exercise 2.** Let  $A$  be a subset of  $X$ . We say that  $A$  is a retract of  $X$  if there is a continuous map  $r : X \rightarrow A$  such that  $r(a) = a$  for all  $a \in A$ . Show that

- (1)  $S^1$  is a retract of  $D^2 - \{0\}$ .
- (2)  $S^1$  is not a retract of  $D^2$ .

**Exercise 3.** Show that the map  $e : \mathbb{R} \rightarrow S^1$  given by  $e(t) = e^{2\pi it}$  does not have a continuous section, i.e., there does not exist a continuous map  $s : S^1 \rightarrow \mathbb{R}$  such that  $e(s(x)) = x$  for all  $x \in S^1$ .

**Exercise 4.** Show that  $\mathbb{R}^n$  is homeomorphic to  $\mathbb{R}^m$  if and only if  $n = m$ .

**Exercise 5.** Let  $X$  be the one point union of two circles. (So  $X$  is the figure “eight” and geometrically looks like 8.) Compute the cohomology groups  $H^q(X, \mathbb{Z})$ .

*(Hint: Let  $A$  and  $B$  be the two circles and let  $P = A \cap B$  be their common point of intersection in  $X$ . Let  $a : A \hookrightarrow X$ ,  $b : B \hookrightarrow X$  and  $p : P \hookrightarrow X$  be the inclusion maps. Consider the short exact sequence*

$$0 \rightarrow \mathbb{Z}_X \rightarrow a_*\mathbb{Z}_A \oplus b_*\mathbb{Z}_B \rightarrow p_*\mathbb{Z}_P \rightarrow 0$$

*of sheaves on  $X$ .)*

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