

Math 5283, HW Set 3, Fall 2006

Due: Wednesday, September 27, 2006

1. Page 72, Problem 3 in Ahlfors.
2. Page 83, Problem 4 in Ahlfors.
3. Page 96, Problem 1 in Ahlfors.
4. Page 97, Problem 6 in Ahlfors.
5. Show that under the stereographic projection the angle between any two curves on the Riemann sphere (with their intersection not at the north pole) equals the angle between the images of the curves in the complex plane.
6. (a) Assume that $w = Tz$ is a linear fractional transformation (also called a Möbius transformation) with a single finite fixed point z_0 . Prove that

$$\frac{1}{w - z_0} = \frac{1}{z - z_0} + h$$

for some $h \neq 0$.

(b) Next assume that $w = Tz$ is a Möbius transformation with two distinct finite fixed points z_1 and z_2 . Prove that

$$\frac{w - z_1}{w - z_2} = k \frac{z - z_1}{z - z_2}$$

for some k .

7. Show that $z \mapsto 1/z$ and $z \mapsto 1 - z$ generate a finite subgroup in the group of Möbius transformations.