

**Math 5283, HW Set 4, Fall 2006**

Due: Wednesday, October 4, 2006

1. Let  $P(z) = (z - a_1)(z - a_2) \cdots (z - a_k)$  and let  $\gamma$  be a closed curve with no roots of  $P$  on  $\gamma$ . Show that

$$\frac{1}{2\pi i} \int_{\gamma} \frac{P'(z)}{P(z)} dz = n(\gamma, a_1) + \cdots + n(\gamma, a_k).$$

2. Page 108, Problem 6 in Ahlfors.  
3. Page 120, Problem 2 in Ahlfors.  
4. Page 123, Problem 1 in Ahlfors.  
5. Page 123, Problem 2 in Ahlfors.  
6. Page 123, Problem 4 in Ahlfors.  
7. Let  $f(z)$  be an analytic function on the unit disk  $|z| \leq 1$  and set

$$d := \sup_{|z| \leq 1, |w| \leq 1} |f(z) - f(w)|.$$

Prove that  $f'(0) \leq d/2$ . (Hint: Start by showing

$$2f'(0) = \frac{1}{2\pi i} \int_C \frac{f(\zeta) - f(-\zeta)}{\zeta^2} d\zeta,$$

where  $C$  is any circle of radius  $r$ ,  $0 < r < 1$  around the origin.)