Exam 3, Math 2163, Sec 001 & 003, Fall 2005

Name : _____ **Section :** 1(7:30) 3(9:30) circle one.

Each problem below requires some arguments and/or computations. You should clearly and completely show all your work and reasons to receive full credit. You may use any statement from the book and the class as long as you clearly state what fact you are using.

1. Let $\mathbf{F}(x, y, z) = x^2\mathbf{i} - xy\mathbf{j} + z\mathbf{k}$. Compute the following:

(a) (3 pt) $\operatorname{div}\mathbf{F}$.

(b) (3 pts) $\operatorname{curl}\mathbf{F}$.

2.(3 pts) Determine whether the following vector field defined on $\mathbb{R}^2$ is conservative.

$$\mathbf{F}(x, y) = xe^y\mathbf{i} + ye^x\mathbf{j}$$

3.(3 pts) Let $\mathbf{F}(x, y) = y\mathbf{i} + (x + 2y)\mathbf{j}$. Find a function f such that $\mathbf{F} = \nabla f$ and use it to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the upper semicircle that starts at $(0, 1)$ and ends at $(2, 1)$.

4.(3 pts) Use Green's theorem to evaluate the following line integral, where C is the circle $x^2 + y^2 = 9$ oriented counterclockwise.

$$\int_C (3y - e^{\sin x})\,dx + (7x + \sqrt{y^4 + 1})\,dy$$

5.(3 pts) Let $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{r}$, where S is part of the graph of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$, and has upward orientation.

6.(2 pts) Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = |\mathbf{r}|$. Compute $\nabla(1/r)$ in terms of r and $\mathbf{r}$. (This means your answer should not involve x, y, z.)