TORA VIII Abstracts

All lectures are in MSCS 101.

Ben Brubaker (University of Minnesota)
Title: Hecke algebras everywhere
(Colloquium lecture)

Abstract: Iwahori-Hecke algebras are generalizations of some of the most important group algebras – those of Coxeter groups like the symmetric group or dihedral groups. So they appear all over mathematics where actions of these groups are involved. They appeared in Jones' work on polynomial invariants for knots, on Kazhdan and Lusztig's work in computing cohomology of nice classes of varieties, and in the study of representations of Lie groups over finite and local fields. In the latter case, they can turn problems about infinite dimensional representations into beautiful algebraic combinatorics. We'll define them and discuss some of their remarkable appearances, and at the end describe some recent joint work (with Bump and Friedberg) on computing matrix coefficients of certain group representations using Hecke algebras. No prior knowledge about Hecke algebras will be assumed.

Ben Brubaker (University of Minnesota)
Title: Ice models: Statistical mechanics and automorphic forms
(TORA lecture)

Abstract: Symmetric functions appear in automorphic forms when we try to compute local matrix coefficients. And one tool for studying symmetric functions are statistical mechanical models and the Yang-Baxter equation. I'll describe this process with some simple examples and then discuss recent joint work with Buciumas and Bump that relates Whittaker functions to statistical mechanics, leading to some surprising connections to quantum groups.

Shuyang Cheng (University of Michigan)
Title: L-functions and functoriality, some recent approaches

Abstract: Automorphic L-functions are closely related to the functoriality conjecture, in particular various methods have been found to establish analytic properties of automorphic L-functions. In this talk I will survey some of the more recent approaches to study automorphic L-functions.

Ameya Pitale (University of Oklahoma)
Title: Integral representation and critical L-values for holomorphic forms on $GSp(2n) \times GL(1)$

Abstract: In this talk, we will report on recent joint work with Abhishek Saha and Ralf Schmidt on integral representation of the standard L-function for holomorphic vector-valued Siegel modular forms of arbitrary genus and with respect to arbitrary congruence subgroup. A lot of work has been done on this topic by Andrianov, Harris, Sturm, Garrett, Shimura, Piatetski-Shapiro, Rallis and many others. To obtain the most general result, we adopt the adelic approach and obtain the pullback of an Eisenstein series on $GSp(4n)$ to $GSp(2n) \times GSp(2n)$. The innovation is the choice of vectors in the ramified and the archimedean cases allowing us to get explicit formulas. The potential applications are arithmeticity of special values of L-functions as algebraic numbers (normalized by suitable periods), and one can further ask the prime factorization of those algebraic numbers. We will report on the arithmeticity r esults for the genus 2 case, which involves a deeper understanding of the structure of nearly holomorphic modular forms.

Olav Richter (University of North Texas)
Title: The skew-Maass lift

Abstract: I will report on recent joint work with O. Imamoglu and M. Westerholt-Raum. The classical Maass lift is a map from holomorphic Jacobi forms to holomorphic scalar-valued Siegel modular forms. Automorphic representation theory predicts a non-holomorphic and vector-valued analogue, which we will construct in this talk. Specifically, we lift skew-holomorphic Jacobi forms to certain real-analytic, vector-valued Siegel modular forms. As a by-product, we give a complete answer to Kohnen's question on the connection between skew-holomorphic Jacobi forms and real-analytic Siegel modular forms. In addition, our method yields Maass relations in a straightforward way.

Fredoon Shahidi (Purdue University)
Title: L-functions and Monoids

Abstract: I will discuss ideas of Braverman-Kazhdan/Ngo on generalizing the work of Godement-Jacquet on principal L-functions for $GL(n)$. This requires an object which generalizes M(n), the algebra of nxn matrices, a reductive "monoid", a space of Schwartz functions on its subgroup of units together with a Fourier transform, and finally a Poisson summation formula for its globalization. They are to be defined by means of the corresponding representation of the L-group. I will then go through a number of examples and explain these objects whenever possible. These questions are also related to Langlands' "Beyond Endoscopy".

Birgit Speh (Cornell University)
Title: Restrictions of infinite dimensional representations

A representation π of a group G defines a representation of a subgroup G' by restriction. In general irreducibility is not preserved by the restriction. If G is compact, then the restriction $\pi|_{G'}$ is isomorphic to a direct sum of irreducible representations π' of G' with multiplicities $m(\pi,\pi')$ which are studied by using combinatorial techniques. If G' is not compact and the representation π is infinite-dimensional, then generically the restriction $\pi|_{G'}$ is not a direct sum of irreducible representations and we have to consider another notion of multiplicity.

For a continuous representation π of G on a complete, locally convex topological vector space H_π, the space H_π^∞ of C^∞-vectors of H_π is naturally endowed with a Fréchet topology, and (π, H_π) gives rise to a continuous representation π^∞ of G on H_π^∞. Given another continuous representation π' of the subgroup G', we consider the space of continuous G'-intertwining operators

$$\mathrm{Hom}_{G'}(\pi^\infty|_{G'}, (\pi')^\infty).$$

The dimension $m(\pi,\pi')$ of this space yields important information of the restriction of π to G' and is called the *multiplicity* of π'. For example if π' is the trivial representation $\mathbb{C}$ then $m(\pi,\mathbb{C})$ is dimension of the space of G'–invariant linear forms on π^∞.

In this talk I will discuss the multiplicities $m(\pi,\pi')$ for a family of irreducible representations of $G = O(n+1,1)$ and $G' = O(n,1)$ and time permitting the dimension $m(\pi_1\otimes\pi_2\otimes\pi_3,\mathbb{C})$ of the space of invariant trilinear forms on induced representations of $PGL(2,\mathbb{R})$. I will outline the connection of these results with the famous conjectures by B. Gross and D. Prasad.

This is joint work with R.Gomez (Cornell) and T.Kobayashi (Tokyo).

Ben Wyser (Univeristy of Illinois Urbana-Champaign)
Title: Singularities of K-orbit closures and interval pattern avoidance

The local structure of symmetric subgroup orbit closures on the flag variety is of importance in the theory of Harish-Chandra modules for real Lie groups. Thanks largely to work of McGovern and McGovern-Trapa, certain local properties of such orbit closures, such as smoothness, are known to be characterized by pattern avoidance in many cases. However, there are more refined local properties which cannot be characterized in the same way. I will describe a generalization of pattern avoidance, which we call interval pattern avoidance, which governs all reasonable local properties of K-orbit closures in the case where $G = GL_{p+q}$ and $K = GL_p \times GL_q$. Although combinatorial in nature, this result follows from underlying geometry: An interval pattern embedding implies an isomorphism of two "slices" of the corresponding orbit closures. This work is joint with Alexander Woo and Alexander Yong.