Math 2233
Homework Set 4

1. Verify that each of the following differential equations is exact and then find the general solution.

(a) $2xy\,dx + \left(x^2+1\right)dy = 0$

(b) $3x^2y\,dx + \left(x^3+1\right)dy = 0$

(c) $y(y+2x)dx + x(2y+x)dy = 0$

(d) $y\cos(xy)\,dx + x\cos(xy)\,dy = 0$

2. Solve the following initial value problems.

(a) $(x - y\cos(x)) - \sin(x)y' = 0$, $y\left(\frac{\pi}{2}\right) = 1$

(b) $x^2 + y^2 + 2xyy' = 0$, $y(1) = 1$

3. Find an integrating factor for each of the following differential equations and obtain the general solution.

(a) $y + (y-x)y' = 0$

(b) $x^2 + y^2 + x + yy' = 0$

(c) $2y^2 + (2x+3xy)y' = 0$

(d) $xy - x^2y' = 0$

5. Solve the following first order differential equations using the substitution $u = y/x$.

(a) $xy' - y = \sqrt{xy}$

(b) $y' = \dfrac{y^2+xy}{x^2}$, $y(1) = 1$

(c) $3xyy' + x^2 + y^2 = 0$

6. Find a substitution that provides a solution to the following differential equations.

(a) $xy' + y = (xy)^3$

(b) $(x+y)y' = (2x+2y) - 3$