Math 3013
Homework Set 1

Problems from §1.1 (pg. 15 - 17 of text): 1,9,31,35
Problems from §1.2 (pg. 31 - 33 of text): 1,3,22,23,25,27,33
Problems from §1.3 (pg. 46 - 48 of text): 1,3,7,9,11,13,14,19,21

1. (Problem 1.1.1 and 1.1.3 in text). Let $\mathbf{v} = [2, -1]$ and $\mathbf{w} = [-2, -3]$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$ and then draw coordinate axes and sketch, using your answers the vectors $\mathbf{v}, \mathbf{w}, \mathbf{v} + \mathbf{w}$, and $\mathbf{v} - \mathbf{w}$.

(a) $\mathbf{v} = [2, -1]$, $\mathbf{w} = [-3, -2]$.

(b) $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, where $\mathbf{i} = [1, 0, 0]$, $\mathbf{j} = [0, 1, 0]$, $\mathbf{k} = [0, 0, 1]$ are the standard basis vectors of $\mathbb{R}^3$.

2. (Problem 1.1.9 in text). Let $\mathbf{u} = [1, 2, 1, 0]$, $\mathbf{v} = [-2, 0, 1, 6]$ and $\mathbf{w} = [3, -5, 1, -2]$. Compute $\mathbf{u} - 2\mathbf{v} + 4\mathbf{w}$.

3. (Problem 1.1.31 in text). Find the vector which, when translated, represents geometrically an arrow reaching from the point $(-1, 3)$ to the point $(4, 2)$ in $\mathbb{R}^2$.

4. (Problems 1.2.1 and 1.2.3 in text). Let $\mathbf{u} = [-1, 3, 4]$ and $\mathbf{v} = [2, 1, -1]$. Compute $\|-\mathbf{u}\|$ and $\|\mathbf{v} + \mathbf{u}\|$.

5. (Problem 1.2.22 in text). Compute the angle between $[1, -1, 2, 3, 0, 4]$ and $[7, 0, 1, 3, 2, 4]$ in $\mathbb{R}^6$.

6. (Problem 1.2.23 in text) Prove that $(2, 0, 4)$, $(4, 1, -1)$ and $(6, 7, 7)$ are the vertices of a right triangle in $\mathbb{R}^3$.

7. (Problems 1.2.25 and 1.2.27 in text). Classify the vectors as parallel, perpendicular, or neither. If they are parallel, state whether they have the same or opposite directions.

(a) $[-1, 4]$ and $[8, 2]$.

(b) $[3, 2, 1]$ and $[-9, -6, -3]$.

8. (Problem 1.2.22 in text). Find the distance between the points $(2, -1, 3)$ and $(4, 1, -2)$ in $\mathbb{R}^3$.

9. (Problems 1.3.1, 1.3.3, 1.3.7, 1.3.11,1.3.13, and 1.3.14 in text). Let

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 0 & -1 \end{bmatrix} \quad , \quad \mathbf{B} = \begin{bmatrix} 4 & 1 & -2 \\ 5 & -1 & 3 \end{bmatrix} \quad , \quad \mathbf{C} = \begin{bmatrix} 2 & -1 \\ 0 & 6 \\ -3 & 2 \end{bmatrix} \quad , \quad \mathbf{D} = \begin{bmatrix} -4 & 2 \\ 3 & 5 \\ -1 & -3 \end{bmatrix}$$

(a) $3\mathbf{A}$

(b)$\mathbf{A} + \mathbf{B}$

(c) $\mathbf{AB}$

(d) $\mathbf{A}^2$

(e) $(2\mathbf{A} - \mathbf{B})\,\mathbf{D}$

(f) $\mathbf{ADB}$

10. (Problem 1.3.19 in text). Consider the row and column vectors

$$\mathbf{x} = [-2, 3, -1] \quad , \quad \mathbf{y} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

Compute the matrix products $\mathbf{xy}$ and $\mathbf{yx}$.

11. (Problem 1.3.21 in text). Mark the following statements True or False.

a. If $\mathbf{A} = \mathbf{B}$, then $\mathbf{AC} = \mathbf{BC}$.

b. If $\mathbf{AC} = \mathbf{BC}$, then $\mathbf{A} = \mathbf{B}$.

c. If $\mathbf{AB} = \mathbf{0}$, then $\mathbf{A} = \mathbf{0}$ or $\mathbf{B} = \mathbf{0}$.

d. If $\mathbf{A} + \mathbf{C} = \mathbf{B} + \mathbf{C}$, then $\mathbf{A} = \mathbf{B}$.

e. If $\mathbf{A}^2 = \mathbf{I}$, then $\mathbf{A} = \pm\mathbf{I}$.

f. If $\mathbf{B} = \mathbf{A}^2$ and if $\mathbf{A}$ is an $n \times n$ matrix and symmetric, then $b_{ii} \geq 0$ for $i = 1, 2. \ldots, n$.

g. If $\mathbf{AB} = \mathbf{C}$ and if two of the matrices are square, then so is the third.

h. If $\mathbf{AB} = \mathbf{C}$ and if $\mathbf{C}$ is a column vector then so is $\mathbf{B}$.

i. If $\mathbf{A}^2 = \mathbf{I}$, then $\mathbf{A}^n = \mathbf{I}$ for all integers $n \geq 2$.

j. If $\mathbf{A}^2 = \mathbf{I}$, then $\mathbf{A}^n = \mathbf{I}$ for all even integers $n \geq 2$.