

Math 3013
Problem Set 5

1. Determine which of the following mappings are linear transformations.

(a) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 : T([x_1, x_2, x_3]) = [x_1 + x_2, x_1 - 3x_2]$

(b) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4 : T([x_1, x_2, x_3]) = [0, 0, 0, 0]$

(c) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4 : T([x_1, x_2, x_3]) = [1, 1, 1, 1]$

(d) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 : T([x_1, x_2]) = [x_1 - x_2, x_2 + 1, 3x_1 - 2x_2]$

2. For each of the following, assume T is a linear transformation, from the data given, compute the specified value.

(a) Given $T([1, 0]) = [3, -1]$, and $T([0, 1]) = [-2, 5]$, find $T([4, -6])$.

(b) Given $T([1, 0, 0]) = [3, 1, 2]$, $T([0, 1, 0]) = [2, -1, 4]$, and $T([0, 0, 1]) = [6, 0, 1]$, find $T([2, -5, 1])$.

3. Find the standard matrix representations of the following linear transformations.

(a) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 : T([x_1, x_2]) = [x_1 + x_2, x_1 - 3x_2, x_1 + x_2]$

(b) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 : T([x_1, x_2, x_3]) = [x_1 + x_2 + x_3, x_1 + x_2]$

(c) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 : T([x_1, x_2, x_3]) = [x_1 + x_2 + x_3, 2x_1 + 2x_2 + 2x_3]$

4. For each of the linear transformations $T : \mathbb{R}^m \rightarrow \mathbb{R}^n$ in Problem 3, determine

$$\text{Range}(T) := \{\mathbf{y} \in \mathbb{R}^n \mid \mathbf{y} = T(\mathbf{x}) \text{ for some } \mathbf{x} \in \mathbb{R}^m\}$$

and

$$\text{Kernel}(T) := \{\mathbf{x} \in \mathbb{R}^m \mid T(\mathbf{x}) = \mathbf{0}_{\mathbb{R}^n}\} \quad .$$

5. If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is defined by $T([x_1, x_2]) = [2x_1 + x_2, x_1, x_1 - x_2]$ and $T' : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is defined by $T'([x_1, x_2, x_3]) = [x_1 - x_2 + x_3, x_1 + x_2]$, find the standard matrix representation for the linear transformation $T' \circ T$ that carries \mathbb{R}^2 into \mathbb{R}^2 . Find a formula for $(T' \circ T)([x_1, x_2])$.