Math 3013
Problem Set 8

1. For each matrices below

- Determine if the matrix is diagonalizable.
- If the matrix is diagonalizable, find a diagonal matrix $\mathbf{D}$ and an invertible matrix $\mathbf{C}$ such that $\mathbf{C}^{-1}\mathbf{AC} = \mathbf{D}$.

(a) $\begin{pmatrix} 5 & -1 \\ 2 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 4 \end{pmatrix}$

2. Let $B = \{[1,-1],[1,1]\}$ and $B' = \{[1,2],[2,1]\}/$

- Show B and B' are both bases for $\mathbb{R}^2$.
- Find the change-of-basis matrix that converts vectors $[x_1, x_2]$ in $\mathbb{R}^2$ to their coordinate vectors with respect to the basis B.
- Find the change-of-basis matrix that converts coordinate vectors with respect to B' to coordinate vectors with respect to B.

3. Given that $B = \{[1,0,1],[0,1,1]],[0,0,1]\}$ and $B' = \{[1,1,-1],[1,1,0],[1,0,0]\}$ are two bases for $\mathbb{R}^3$. Find the change of basis matrix that converts coordinate vectors with respect to B' to coordinate vectors with respect to B.

4. Find the change-of-basis matrix that converts vectors $[x_1, x_2]$ to their coordinate vectors with respect to a basis consisting of the eigenvectors of $\mathbf{A} = \begin{pmatrix} 5 & -1 \\ 2 & 2 \end{pmatrix}$. (Hint: Use the results of problem 1 (a).)