

Math 3013
Homework Set 8

Problems from §4.1 (pgs. 248-249 of text): 6,7,8,9,12,19

Problems from §4.2 (pgs. 261-263 of text): 1,3,5,7,11,15,17,19,21

Problems from §4.3 (pgs. 271-273 of text): 1,3,5,7,9,15,17,24,29

1. (Problems 4.1.6, 4.1.7, 4.1.8, and 4.1.9 in text) Calculate the following determinants.

$$(a) \begin{vmatrix} 1 & 4 & -2 \\ 5 & 13 & 0 \\ 2 & -1 & 3 \end{vmatrix}$$

$$(b) \begin{vmatrix} 2 & -5 & 3 \\ 1 & 3 & 4 \\ -2 & 3 & 7 \end{vmatrix}$$

$$(c) \begin{vmatrix} 1 & -2 & 7 \\ 0 & 1 & 4 \\ 1 & 0 & 3 \end{vmatrix}$$

$$(d) \begin{vmatrix} 2 & -1 & 1 \\ -1 & 0 & 3 \\ 2 & 1 & -4 \end{vmatrix}$$

2. (Problem 4.1.12 in text) Show by direct calculation that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = - \begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. (Problem 4.1.19 in text) Mark each of the following *True* or *False*.

- (a) The determinant of a 2×2 matrix is a vector.
- (b) If two rows of a 3×3 matrix are interchanged, the sign of the determinant is changed.
- (c) The determinant of a 3×3 matrix is zero if two rows of the matrix are parallel vectors in \mathbb{R}^3 .
- (d) In order for the determinant of a 3×3 matrix to be zero, two rows must be parallel vectors in \mathbb{R}^3 .
- (e) The determinant of a 3×3 matrix is zero if the points in \mathbb{R}^3 lie in a plane.
- (f) The determinant of a 3×3 matrix is zero if the points in \mathbb{R}^3 lie in a plane through the origin.
- (g) The parallelogram in \mathbb{R}^2 determined by non-zero vectors \mathbf{a} and \mathbf{b} is a square if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.
- (h) The box in \mathbb{R}^3 determined by vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} is a cube if and only if $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} = \mathbf{b} \cdot \mathbf{c} = 0$ and $\mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b} = \mathbf{c} \cdot \mathbf{c}$.
- (i) If the angle between two vectors \mathbf{a} and \mathbf{b} in \mathbb{R}^3 is $\pi/4$, then $\|\mathbf{a} \times \mathbf{b}\| = |\mathbf{a} \cdot \mathbf{b}|$.
- (j) For any vector \mathbf{a} in \mathbb{R}^3 we have $\|\mathbf{a} \times \mathbf{a}\| = \|\mathbf{a}\|^2$.

4. (Problems 4.2.1, 4.2.3, 4.2.5, and 4.2.7 in text) Compute the determinants of the following matrices.

5. (Problem 4.2.11 in text). Find the cofactor of 5 for the matrix $\begin{bmatrix} 1 & 0 & 6 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

6. (Problems 4.2.15, 4.2.17, and 4.2.19 in text). Suppose \mathbf{A} is a 3×3 matrix such that $\det(\mathbf{A}) = 2$. Compute the following.

(a) $\det(\mathbf{A}^2)$

(b) $\det(3\mathbf{A})$

(c) $\det(\mathbf{A}^{-1})$

7. (Problem 4.2.21 in text) Mark each of the following True or False.

(a) The determinant $\det(\mathbf{A})$ is defined for any matrix \mathbf{A} .

(b) The determinant $\det(\mathbf{A})$ is defined for any square matrix \mathbf{A} .

(c) The determinant of a square matrix is a scalar.

(d) If matrix \mathbf{A} is multiplied by a scalar c , then the determinant of the resulting matrix is $c \cdot \det(\mathbf{A})$.

(e) If an $n \times n$ matrix \mathbf{A} is multiplied by a scalar c , then the determinant of the resulting matrix is $c^n \cdot \det(\mathbf{A})$.

(f) For every square matrix \mathbf{A} , we have $\det(\mathbf{A}\mathbf{A}^T) = \det(\mathbf{A}^T\mathbf{A}) = (\det(\mathbf{A}))^2$

(g) If two rows and also two columns of square matrix are interchanged, the determinant changes sign.

(h) The determinant of an elementary matrix is nonzero.

(i) If $\det(\mathbf{A}) = 2$ and $\det(\mathbf{B}) = 3$, then $\det(\mathbf{A} + \mathbf{B}) = 5$.

(j) If $\det(\mathbf{A}) = 2$ and $\det(\mathbf{B}) = 3$, then $\det(\mathbf{A}\mathbf{B}) = 6$.

8. (Problems 4.3.1, 4.3.3, 4.3.5, 4.3.7, and 4.3.9 in text). Compute the determinants of the following matrices.

(a) $\mathbf{A} = \begin{bmatrix} 2 & 3 & -1 \\ 5 & -7 & 1 \\ -3 & 2 & -1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 5 & 2 & 4 & 0 \\ 2 & -3 & -1 & 2 \\ 3 & -4 & 3 & 7 \\ 1 & -1 & 0 & 1 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -1 & 2 \\ 0 & 0 & -3 & 2 & 4 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$

$$(d) \mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 2 & 0 & -3 \\ 0 & -2 & 1 & 0 & 0 \\ 5 & -3 & 2 & 0 & 0 \\ -3 & 4 & 0 & 0 & 0 \end{bmatrix}$$

$$(e) \mathbf{A} = \begin{bmatrix} 2 & -1 & 3 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ -5 & 2 & 6 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -2 & 8 \end{bmatrix}$$

9. (Problems 4.3.15 and 4.3.17 in text) Use the corollary to Theorem 4.6 to find \mathbf{A}^{-1} if \mathbf{A} is invertible.

$$(a) \mathbf{A} = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$$

$$(b) \mathbf{A} = \begin{bmatrix} 3 & 0 & 4 \\ -2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

10. (Problems 4.3.24 and 4.3.29 in text). Solve the following systems of linear equations using Cramer's Rule.

$$(a) \begin{aligned} x_1 - 2x_2 &= 1 \\ 3x_1 + 4x_2 &= 3 \end{aligned}$$

$$(b) \begin{aligned} x_1 + 2x_2 - x_3 &= -2 \\ 2x_1 + x_2 + x_3 &= 0 \\ 3x_1 - x_2 + 5x_3 &= 1 \end{aligned}$$