

Math 3013
Problem Set 9

1. Find a basis for the orthogonal complement W^\perp of each of the subspaces given below

(a) $W = \text{span}([1, 1, 0], [1, 0, 1])$ in \mathbb{R}^3

(b) $W =$ solution set of $x + y - z = 0$

2. For each vector \mathbf{v} and subspace W below, determine the orthogonal decomposition $\mathbf{v} = \mathbf{v}_W + \mathbf{v}_\perp$ of \mathbf{v} with respect to W .

(a) $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, W = \text{span} \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right)$

(b) $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 5 \\ 6 \end{bmatrix}, W = \text{span} \left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right)$

3. For each set of linearly independent vectors below, apply the Gram-Schmidt process to obtain an orthogonal basis for the subspace generated by the vectors.

(a) $\left\{ \begin{bmatrix} 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$