

MATH 3613  
Homework Set 1

1. Prove that “not- $Q \Rightarrow$  not- $P$ ” implies “ $P \Rightarrow Q$ ”
2. Prove that if  $m$  and  $n$  are even integers, then  $n + m$  is an even integer.
3. Prove that if  $n$  is an odd integer, then  $n^2$  is an odd integer.
4. Prove that if  $n$  is an integer and  $n^2$  is odd, then  $n$  is odd.
5. Prove, by the contrapositive method, that if  $c$  is an odd integer then the equation  $n^2 + n - c = 0$  has no integer solution.
6. Prove, by mathematical induction, that if  $n \geq 5$  then  $2^n > n^2$ .
7. Prove by the contrapositive method that if  $c$  is an odd integer, there the equation  $n^2 + n + c = 0$  has no integer solution for  $n$ .
8. Prove by mathematical induction that

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \forall n \in \mathbb{Z}^+ .$$

9. Prove the following identities.

- (a)  $B \cap (C \cup D) = (B \cap C) \cup (B \cap D)$
- (b)  $B \cup (C \cap D) = (B \cup C) \cap (B \cup D)$
- (c)  $C = (C - A) \cup (C \cap A)$

10. Describe each set in set-builder notation:

- (a) All positive real numbers.
- (b) All negative irrational numbers.
- (c) All points in the coordinate plane with rational first coordinate.
- (d) All negative even integers greater than  $-50$ .

11. Which of the following sets are nonempty?

- (a)  $\{r \in \mathbb{Q} \mid r^2 = 2\}$
- (b)  $\{r \in \mathbb{R} \mid r^2 + 5r - 7 = 0\}$
- (c)  $\{t \in \mathbb{Z} \mid 6t^2 - t - 1 = 0\}$

12. Is  $B$  is a subset of  $C$  when

(a)  $B = \mathbb{Z}$  and  $C = \mathbb{Q}$ ?

(b)  $B =$  all solutions of  $x^2 + 2x - 5 = 0$  and  $C = \mathbb{Z}$ ?

(c)  $B = \{a, b, 7, 9, 11, -6\}$  and  $C = \mathbb{Q}$ ?

13. In each part find  $B - C$ ,  $B \cap C$ , and  $B \cup C$ .

(a)  $B = \mathbb{Z}$  and  $C = \mathbb{Q}$ .

(b)  $B = \mathbb{R}$  and  $C = \mathbb{Q}$ .

(c)  $B = \{a, b, c, 1, 2, 3, 4, 5, 6\}$  and  $C = \{a, c, e, 2, 4, 6, 8\}$ .

14. Let  $A, B$  be subsets of  $U$ . Prove De Morgan's laws:

(a)  $U - (A \cap B) = (U - A) \cup (U - B)$

(b)  $U - (A \cup B) = (U - A) \cap (U - B)$

15.

(a) Give an example of a function that is injective but not surjective.

(b) Give an example of a function that is surjective but not injective.

16. Prove that  $f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^3$  is injective.

17. Prove that  $f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = -3x + 5$  is surjective.

18. Let  $B$  and  $C$  be nonempty sets. Prove that the function

$$f : B \times C \rightarrow C \times B$$

given by  $f(x, y) = (y, x)$  is a bijection.