

Cells, Orbits, and Weyl Group Representations

Below we give a series of tables for the various blocks of irreducible Harish-Chandra modules of the exceptional groups. Here is how the data is organized. Each row in each table corresponds to a particular cell in a particular block of an exceptional group. In cases where the adjoint group and the simply connected groups produce the same table we present only the table for the simply connected group. Similarly, whenever the same table results for the split and compact inner classes, we present only the table for the split inner class. The data in the columns of each table is as follows:

- # the cell number as enumerated in the output of the atlas `extract-cells` command
- $|C|$ the cardinality of the cell (i.e. the number of irreducible representations in the cell)
- \mathcal{O}_C the special orbit attached to the cell - computed by determining the minimal Richardson orbits containing the cell's orbit and the minimal Richardson orbits containing the orbit of the corresponding dual cell)
- σ_{spec} the special representation attached to the cell - determined by applying the Springer correspondence to \mathcal{O}_C
- $\#Prim(C)$ the number of primitive ideals in the cell (the number of distinct subcells in the τ_∞ partitioning of the cell)
- $\mathcal{P}_{\tau_\infty}$ the τ_∞ partitioning of the cell written as an actual partition of $|C|$. E.g. $1^{20}2^{50}3^{10}$ means the τ_∞ partitioning of the cell decomposes it into 20 singlets, 50 doublets, and 10 triplets of representations sharing the same primitive ideal.
- W -rep the representation of the Weyl group carried by the cell. This is the coherent continuation representation inferred from the W -graph structure of the cell. It was computed by using the output of `extract-cells` to explicitly construct the (restriction of the) coherent continuation representation to the cell, computing the character of this representation by taking the traces of representatives of each conjugacy class in W , and then writing this character into a sum irreducible characters.

The last series of table contains the analogous results for the complex exceptional groups, excluding $E_8(\mathbb{C})$. The W -representations appearing in these tables have yet to be verified by computation.

1. G_2

11 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	G_2	5	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	5	$G_2(a_1)$	4	$\phi_{2,1}$	2	2^13^1	$\phi_{2,1} + \phi_{2,2} + \phi'_{1,3}$
2	5	$G_2(a_1)$	4	$\phi_{2,1}$	2	2^13^1	$\phi_{2,1} + \phi_{2,2} + \phi''_{1,3}$
3	1	0	1	$\phi_{1,6}$	1	1^1	$\phi_{1,6}$

2. F_4

12 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	8	A_2	6	$\phi'_{8,9}$	8	1^8	$\phi'_{8,9}$
1	6	A_1	3	$\phi_{4,13}$	4	1^22^2	$\phi_{4,13} + \phi'_{2,16}$
2	1	0	1	$\phi_{1,24}$	1	1^1	$\phi_{1,24}$

21 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	F_4	16	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	6	$F_4(a_1)$	15	$\phi_{4,1}$	4	1^22^2	$\phi_{4,1} + \phi'_{2,4}$
2	8	C_3	13	$\phi'_{8,3}$	8	1^8	$\phi'_{8,3}$

22 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$F4$	16	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	6	$F4(a1)$	15	$\phi_{4,1}$	4	$1^2 2^2$	$\phi_{4,1} + \phi''_{2,4}$
3	6	$F4(a1)$	15	$\phi_{4,1}$	4	$1^2 2^2$	$\phi_{4,1} + \phi'_{2,4}$
4	9	$F4(a2)$	14	$\phi_{9,2}$	9	1^9	$\phi_{9,2}$
5	9	$F4(a2)$	14	$\phi_{9,2}$	9	1^9	$\phi_{9,2}$
11	9	$F4(a2)$	14	$\phi_{9,2}$	9	1^9	$\phi_{9,2}$
12	8	$C3$	13	$\phi'_{8,3}$	8	1^8	$\phi'_{8,3}$
2	8	$B3$	12	$\phi''_{8,3}$	8	1^8	$\phi''_{8,3}$
6	8	$B3$	12	$\phi''_{8,3}$	8	1^8	$\phi''_{8,3}$
7	8	$B3$	12	$\phi''_{8,3}$	8	1^8	$\phi''_{8,3}$
8	8	$B3$	12	$\phi''_{8,3}$	8	1^8	$\phi''_{8,3}$
9	57	$F4(a3)$	11	$\phi_{12,4}$	12	$2^3 3^1 5^3 6^4 9^1$	$\phi_{12,4} + \phi_{16,5} + 2\phi''_{9,6} + \phi''_{6,6} + \phi''_{4,7} + \phi''_{1,12}$
13	47	$F4(a3)$	11	$\phi_{12,4}$	12	$5^4 4^4 3^3 2^1$	$\phi_{12,4} + \phi_{16,5} + \phi''_{9,6} + \phi'_{6,6} + \phi''_{4,7}$
14	72	$F4(a3)$	11	$\phi_{12,4}$	12	$4^6 6^2 9^4$	$\phi_{12,4} + 2\phi_{16,5} + \phi'_{9,6} + \phi''_{9,6} + \phi''_{6,6} + \phi_{4,8}$
18	8	$A2$	6	$\phi'_{8,9}$	8	1^8	$\phi'_{8,9}$
10	8	$A2$	5	$\phi''_{8,9}$	8	1^8	$\phi''_{8,9}$
15	8	$A2$	5	$\phi''_{8,9}$	8	1^8	$\phi''_{8,9}$
16	8	$A2$	5	$\phi''_{8,9}$	8	1^8	$\phi''_{8,9}$
21	8	$A2$	5	$\phi''_{8,9}$	8	1^8	$\phi''_{8,9}$
17	9	$A1 + \bar{A}1$	4	$\phi_{9,10}$	9	1^9	$\phi_{9,10}$
19	9	$A1 + \bar{A}1$	4	$\phi_{9,10}$	9	1^9	$\phi_{9,10}$
20	9	$A1 + \bar{A}1$	4	$\phi_{9,10}$	9	1^9	$\phi_{9,10}$
22	6	$\bar{A}1$	3	$\phi_{4,13}$	4	$1^2 2^2$	$\phi_{4,13} + \phi''_{2,16}$
23	6	$\bar{A}1$	3	$\phi_{4,13}$	4	$1^2 2^2$	$\phi_{4,13} + \phi''_{2,16}$
24	1	0	1	$\phi_{1,24}$	1	1^1	$\phi_{1,24}$

3. E_6 : COMPACT INNER CLASS

11 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	81	$A4$	13	$\phi_{81,6}$	81	1^{81}	$\phi_{81,6}$
2	81	$A3 + A1$	9	$\phi_{81,10}$	81	1^{81}	$\phi_{81,10}$
3	81	$A3 + A1$	9	$\phi_{81,10}$	81	1^{81}	$\phi_{81,10}$
1	24	$2A2$	7	$\phi_{24,12}$	24	1^{24}	$\phi_{24,12}$
4	64	$A2 + A1$	6	$\phi_{64,13}$	64	1^{64}	$\phi_{64,13}$
6	64	$A2 + A1$	6	$\phi_{64,13}$	64	1^{64}	$\phi_{64,13}$
8	45	$A2$	5	$\phi_{30,15}$	30	$1^{15} 2^{15}$	$\phi_{30,15} + \phi_{15,17}$
5	20	$2A1$	3	$\phi_{20,20}$	20	1^{20}	$\phi_{20,20}$
7	20	$2A1$	3	$\phi_{20,20}$	20	1^{20}	$\phi_{20,20}$
9	20	$2A1$	3	$\phi_{20,10}$	20	1^{20}	$\phi_{20,20}$
10	6	$A1$	2	$\phi_{6,25}$	6	1^6	$\phi_{6,25}$
11	6	$A1$	2	$\phi_{6,25}$	6	1^6	$\phi_{6,25}$
12	1	0	1	$\phi_{1,36}$	1	1^1	$\phi_{1,36}$

20 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E6$	21	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
2	24	$D4$	14	$\phi_{24,6}$	24	1^{24}	$\phi_{24,6}$

21 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E6$	21	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
3	6	$E6(a1)$	20	$\phi_{6,1}$	6	1^6	$\phi_{6,1}$
1	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
4	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
5	45	$E6(a3)$	18	$\phi_{30,3}$	30	$1^{15}2^{15}$	$\phi_{30,3} + \phi_{15,4}$
11	45	$E6(a3)$	18	$\phi_{30,3}$	30	$1^{15}2^{15}$	$\phi_{30,3} + \phi_{15,4}$
6	64	$D5(a1)$	17	$\phi_{64,4}$	64	1^{64}	$\phi_{64,4}$
7	64	$D5(a1)$	17	$\phi_{64,4}$	64	1^{64}	$\phi_{64,4}$
12	60	$A4 + A1$	15	$\phi_{60,5}$	60	1^{60}	$\phi_{60,5}$
13	60	$A4 + A1$	15	$\phi_{60,5}$	60	1^{60}	$\phi_{60,5}$
2	24	$D4$	14	$\phi_{24,6}$	24	1^{24}	$\phi_{24,6}$
9	24	$D4$	14	$\phi_{24,6}$	24	1^{24}	$\phi_{24,6}$
8	81	$A4$	13	$\phi_{81,6}$	81	1^{81}	$\phi_{81,6}$
14	81	$A4$	13	$\phi_{81,6}$	81	1^{81}	$\phi_{81,6}$
10	230	$D4(a1)$	12	$\phi_{80,7}$	80	$2^{10}3^{70}$	$\phi_{80,7} + \phi_{90,8} + \phi_{60,8}$
15	150	$D4(a1)$	12	$\phi_{80,7}$	80	$1^{20}2^{50}3^{10}$	$\phi_{80,7} + \phi_{60,8} + \phi_{10,9}$
16	230	$D4(a1)$	12	$\phi_{80,7}$	80	$2^{10}3^{70}$	$\phi_{80,7} + \phi_{90,8} + \phi_{60,8}$
19	81	$A3 + A1$	9	$\phi_{81,10}$	81	1^{81}	$\phi_{81,10}$
21	81	$A3 + A1$	9	$\phi_{81,10}$	81	1^{81}	$\phi_{81,10}$
17	60	$A2 + 2A1$	8	$\phi_{60,11}$	60	1^{60}	$\phi_{60,11}$
18	60	$A2 + 2A1$	8	$\phi_{60,11}$	60	1^{60}	$\phi_{60,11}$
22	60	$A2 + 2A1$	8	$\phi_{60,11}$	60	1^{60}	$\phi_{60,11}$
23	24	$2A2$	7	$\phi_{24,12}$	24	1^{24}	$\phi_{24,12}$
24	64	$A2 + A1$	6	$\phi_{64,13}$	64	1^{64}	$\phi_{64,13}$
25	64	$A2 + A1$	6	$\phi_{64,13}$	64	1^{64}	$\phi_{64,13}$
20	45	$A2$	5	$\phi_{30,15}$	30	$1^{15}2^{15}$	$\phi_{30,15} + \phi_{15,16}$
26	45	$A2$	5	$\phi_{30,15}$	30	$1^{15}2^{15}$	$\phi_{30,15} + \phi_{15,16}$
27	45	$A2$	5	$\phi_{30,15}$	30	$1^{15}2^{15}$	$\phi_{30,15} + \phi_{15,16}$
28	20	$2A1$	3	$\phi_{20,20}$	20	1^{20}	$\phi_{20,20}$
29	20	$2A1$	3	$\phi_{20,20}$	20	1^{20}	$\phi_{20,20}$
30	6	$A1$	2	$\phi_{6,25}$	6	1^6	$\phi_{6,25}$
31	1	0	1	$\phi_{1,36}$	1	1^1	$\phi_{1,36}$

4. E_6 : SPLIT INNER CLASS

02 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
10	24	$2A2$	7	$\phi_{24,12}$	24	1^{24}	$\phi_{24,12}$
1	20	$2A1$	3	$\phi_{20,20}$	20	1^{20}	$\phi_{20,20}$
2	1	0	1	$\phi_{1,36}$	1	1^1	$\phi_{1,36}$

11 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E6$	21	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	6	$E6(a1)$	20	$\phi_{6,1}$	6	1^6	$\phi_{6,1}$
2	6	$E6(a1)$	20	$\phi_{6,1}$	6	1^6	$\phi_{6,1}$
3	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
5	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
7	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
4	45	$E6(a3)$	18	$\phi_{30,3}$	30	$1^{15}2^{15}$	$\phi_{30,3} + \phi_{15,5}$
6	64	$D5(a1)$	17	$\phi_{64,4}$	64	1^{64}	$\phi_{64,4}$
8	64	$D5(a1)$	17	$\phi_{64,4}$	64	1^{64}	$\phi_{64,4}$
9	24	$D4$	14	$\phi_{24,6}$	24	1^{24}	$\phi_{24,6}$
10	81	$A4$	13	$\phi_{81,6}$	81	1^{81}	$\phi_{81,6}$
11	81	$A4$	13	$\phi_{81,6}$	81	1^{81}	$\phi_{81,6}$
12	81	$A3 + A1$	9	$\phi_{81,10}$	81	1^{81}	$\phi_{81,6}$

12 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E6$	21	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	6	$E6(a1)$	20	$\phi_{6,1}$	6	1^6	$\phi_{6,1}$
2	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
3	20	$D5$	19	$\phi_{20,2}$	20	1^{20}	$\phi_{20,2}$
4	45	$E6(a3)$	18	$\phi_{30,3}$	30	$1^{15}2^{15}$	$\phi_{30,3} + \phi_{15,4}$
5	45	$E6(a3)$	18	$\phi_{30,3}$	30	$1^{15}2^{15}$	$\phi_{30,3} + \phi_{15,4}$
15	45	$E6(a3)$	18	$\phi_{30,3}$	30	$1^{15}2^{15}$	$\phi_{30,3} + \phi_{15,4}$
6	64	$D5(a1)$	17	$\phi_{64,4}$	64	1^{64}	$\phi_{64,4}$
7	64	$D5(a1)$	17	$\phi_{64,4}$	64	1^{64}	$\phi_{64,4}$
9	60	$A4 + A1$	15	$\phi_{60,5}$	60	1^{60}	$\phi_{60,5}$
10	60	$A4 + A1$	15	$\phi_{60,5}$	60	1^{60}	$\phi_{60,5}$
11	60	$A4 + A1$	15	$\phi_{60,5}$	60	1^{60}	$\phi_{60,5}$
8	24	$D4$	14	$\phi_{24,6}$	24	1^{24}	$\phi_{24,6}$
12	81	$A4$	13	$\phi_{81,6}$	81	1^{81}	$\phi_{81,6}$
16	81	$A4$	13	$\phi_{81,6}$	81	1^{81}	$\phi_{81,6}$
13	230	$D4(a1)$	12	$\phi_{80,7}$	80	$2^{10}3^{70}$	$\phi_{80,7} + \phi_{90,8} + \phi_{60,8}$
14	150	$D4(a1)$	12	$\phi_{80,7}$	80	$1^{20}2^{50}3^{10}$	$\phi_{80,7} + \phi_{60,8} + \phi_{10,9}$
17	230	$D4(a1)$	12	$\phi_{80,7}$	80	$2^{10}3^{70}$	$\phi_{80,7} + \phi_{90,8} + \phi_{60,8}$
18	81	$A3 + A1$	9	$\phi_{81,10}$	81	1^{81}	$\phi_{81,10}$
23	81	$A3 + A1$	9	$\phi_{81,10}$	81	1^{81}	$\phi_{81,10}$
19	60	$A2 + 2A1$	8	$\phi_{60,11}$	60	1^{60}	$\phi_{60,11}$
20	60	$A2 + 2A1$	8	$\phi_{60,11}$	60	1^{60}	$\phi_{60,11}$
22	24	$2A2$	7	$\phi_{24,12}$	24	1^{24}	$\phi_{24,12}$
27	24	$2A2$	7	$\phi_{24,12}$	24	1^{24}	$\phi_{24,12}$
24	64	$A2 + A1$	6	$\phi_{64,13}$	64	1^{64}	$\phi_{64,13}$
25	64	$A2 + A1$	6	$\phi_{64,13}$	64	1^{64}	$\phi_{64,13}$
21	45	$A2$	5	$\phi_{30,15}$	60	$1^{15}2^{15}$	$\phi_{30,15} + \phi_{15,16}$
26	45	$A2$	5	$\phi_{30,15}$	30	$1^{15}2^{15}$	$\phi_{30,15} + \phi_{15,16}$
28	20	$2A1$	3	$\phi_{20,20}$	20	1^{20}	$\phi_{20,20}$
29	20	$2A1$	3	$\phi_{20,20}$	20	1^{20}	$\phi_{20,20}$
30	6	$A1$	2	$\phi_{6,25}$	6	1^6	$\phi_{6,25}$
31	1	0	1	$\phi_{1,36}$	1	1^1	$\phi_{1,36}$

5. E_7 : SIMPLY CONNECTED

12 Block

<i>Block</i>	#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
12	0	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
12	1	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
12	2	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$

13 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W-rep
0	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
2	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
3	756	$A4$	21	$\phi_{420,13}$	420	$1^{84}2^{336}$	$\phi_{420,13} + \phi_{336,14}$
1	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
4	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
5	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
6	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
11	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
9	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
10	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
12	225	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{15}2^{105}$	$\phi_{120,25} + \phi_{105,26}$
15	225	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{15}2^{105}$	$\phi_{120,25} + \phi_{105,26}$
18	77	$A2$	6	$\phi_{56,30}$	56	$1^{35}2^{21}$	$\phi_{56,30} + \phi_{21,33}$
7	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
8	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
13	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
16	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
14	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
17	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
19	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
20	7	$A1$	2	$\phi_{7,46}$	7	1^7	$\phi_{7,46}$
21	7	$A1$	2	$\phi_{7,46}$	7	1^7	$\phi_{7,46}$
22	1	0	1	$\phi_{1,63}$	1	1^1	$\phi_{1,63}$

21 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W-rep
0	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
1	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
2	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$

23 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
1	135	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{105}2^{15}$	$\phi_{120,4} + \phi_{15,7}$
10	105	$A6$	35	$\phi_{105,6}$	105	1^{105}	$\phi_{105,6}$
2	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
4	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
5	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$	$\phi_{405,8} + \phi_{216,9}$
11	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$	$\phi_{405,8} + \phi_{216,9}$
12	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}	$\phi_{210,10}$
6	756	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{84}2^{336}$	$\phi_{420,10} + \phi_{336,11}$
13	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$
7	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$	$\phi_{420,13} + \phi_{84,15}$
14	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$	$\phi_{420,13} + \phi_{84,15}$
15	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
3	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$
8	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$
9	875	$D4(a1)$	16	$\phi_{315,16}$	315	$2^{70}3^{245}$	$\phi_{315,16} + \phi_{280,17} + \phi_{280,18}$
19	665	$D4(a1)$	16	$\phi_{315,16}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,16} + \phi_{280,17} + \phi_{70,18}$
20	875	$D4(a1)$	16	$\phi_{315,16}$	315	$2^{70}3^{245}$	$\phi_{315,16} + \phi_{280,17} + \phi_{280,18}$
16	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
22	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
17	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
21	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
23	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
24	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
25	225	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{15}2^{105}$	$\phi_{120,25} + \phi_{105,26}$
18	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
26	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
27	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
28	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
29	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
30	7	$A1$	2	$\phi_{7,46}$	7	1^7	$\phi_{7,46}$
31	1	0	1	$\phi_{1,63}$	1	1^1	$\phi_{1,63}$

31 Block

<i>Block</i>	#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
31	0	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
31	1	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
31	2	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
31	3	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
31	4	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
31	5	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
31	7	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
31	6	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
31	14	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
31	8	225	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{15}2^{105}$	$\phi_{120,4} + \phi_{105,5}$
31	11	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
31	9	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
31	10	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
31	12	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
31	15	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
31	13	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
31	16	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$

32 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
2	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
3	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
4	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
5	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
13	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
6	225	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{15}2^{105}$	$\phi_{120,4} + \phi_{105,5}$
7	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
9	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
10	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
14	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
8	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
15	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
11	875	$E7(a5)$	34	$\phi_{315,7}$	315	$2^{70}3^{245}$	$\phi_{315,7} + \phi_{280,8} + \phi_{280,9}$
12	665	$E7(a5)$	34	$\phi_{315,7}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,7} + \phi_{280,8} + \phi_{70,9}$
21	875	$E7(a5)$	34	$\phi_{315,7}$	315	$2^{70}3^{245}$	$\phi_{315,7} + \phi_{280,8} + \phi_{280,9}$
16	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
17	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
23	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
18	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$
22	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
29	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
19	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
24	756	$A4$	21	$\phi_{420,13}$	420	$1^{84}2^{336}$	$\phi_{420,13} + \phi_{336,14}$
20	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
25	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
26	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
30	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
27	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
28	135	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{105}2^{15}$	$\phi_{120,25} + \phi_{15,28}$
31	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$

33 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
2	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
3	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
4	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
6	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
5	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
7	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
10	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
11	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
8	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
9	225	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{15}2^{105}$	$\phi_{120,4} + \phi_{105,5}$
12	135	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{105}2^{15}$	$\phi_{120,4} + \phi_{15,7}$
13	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
14	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
15	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
18	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
22	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
16	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
24	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
17	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
23	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
19	105	$A6$	35	$\phi_{105,6}$	105	1^{105}	$\phi_{105,6}$
27	105	$A6$	35	$\phi_{105,6}$	105	1^{105}	$\phi_{105,6}$
20	875	$E7(a5)$	34	$\phi_{315,7}$	315	$2^{70}3^{245}$	$\phi_{315,7} + \phi_{280,8} + \phi_{280,9}$
25	875	$E7(a5)$	34	$\phi_{315,7}$	315	$2^{70}3^{245}$	$\phi_{315,7} + \phi_{280,8} + \phi_{280,9}$
29	665	$E7(a5)$	34	$\phi_{315,7}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,7} + \phi_{280,8} + \phi_{70,9}$
30	665	$E7(a5)$	34	$\phi_{315,7}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,7} + \phi_{280,8} + \phi_{70,9}$
26	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
28	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$	$\phi_{405,8} + \phi_{216,9}$
31	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$	$\phi_{405,8} + \phi_{216,9}$
21	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
32	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
33	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
35	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
42	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
36	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}	$\phi_{210,10}$
37	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}	$\phi_{210,10}$
34	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
39	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
44	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
38	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$
40	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$
45	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$

33 Block cont'd

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W-rep
52	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
53	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
41	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
43	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
46	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
49	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
54	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$	$\phi_{420,13} + \phi_{84,15}$
57	756	$A4$	21	$\phi_{420,13}$	420	$1^{84}2^{336}$	$\phi_{420,13} + \phi_{336,14}$
55	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
56	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
58	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
63	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
47	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
50	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
59	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
60	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
61	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$
62	665	$D4(a1)$	16	$\phi_{315,16}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,16} + \phi_{280,17} + \phi_{70,18}$
64	875	$D4(a1)$	16	$\phi_{315,16}$	315	$2^{70}3^{245}$	$\phi_{315,16} + \phi_{280,17} + \phi_{280,18}$
69	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
70	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
48	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
51	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
65	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
67	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
71	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
72	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
66	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
68	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
73	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
74	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
75	135	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{105}2^{15}$	$\phi_{120,25} + \phi_{15,28}$
76	135	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{105}2^{15}$	$\phi_{120,25} + \phi_{15,28}$
78	135	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{105}2^{15}$	$\phi_{120,25} + \phi_{15,28}$
77	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
79	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
80	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
81	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
82	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
83	7	$A1$	2	$\phi_{7,46}$	7	1^7	$\phi_{7,46}$
84	1	0	1	$\phi_{1,63}$	1	1^1	$\phi_{1,63}$

6. E_7 : ADJOINT

12 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W-rep
0	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
1	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
2	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$

13 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
1	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$	$\phi_{420,13} + \phi_{84,15}$
2	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
3	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
6	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
7	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
5	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
8	225	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{15}2^{105}$	$\phi_{120,25} + \phi_{105,26}$
11	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
4	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
9	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
10	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
12	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
13	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
14	7	$A1$	2	$\phi_{7,46}$	7	1^7	$\phi_{7,46}$
15	1	0	1	$\phi_{1,63}$	1	1^1	$\phi_{1,63}$
16	1	0	1	$\phi_{1,63}$	1	1^1	$\phi_{1,63}$

21 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
1	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
2	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$

23 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	$\mathcal{P}_{\tau_\infty}^t$	W -rep
0	21	$E6$	41	$\phi_{21,3}$	21	1^{21}		$\phi_{21,3}$
1	135	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{105}2^{15}$		$\phi_{120,4} + \phi_{15,7}$
10	105	$A6$	35	$\phi_{105,6}$	105	1^{105}		$\phi_{105,6}$
2	189	$D5$	33	$\phi_{189,7}$	189	1^{189}		$\phi_{189,7}$
4	189	$D5$	33	$\phi_{189,7}$	189	1^{189}		$\phi_{189,7}$
5	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$		$\phi_{405,8} + \phi_{216,9}$
11	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$		$\phi_{405,8} + \phi_{216,9}$
12	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}		$\phi_{210,10}$
6	756	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{84}2^{336}$		$\phi_{420,10} + \phi_{336,11}$
13	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}		$\phi_{512,11} + \phi_{512,12}$
7	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$		$\phi_{420,13} + \phi_{84,15}$
14	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$		$\phi_{420,13} + \phi_{84,15}$
15	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}		$\phi_{378,14}$
3	105	$D4$	18	$\phi_{105,15}$	105	1^{105}		$\phi_{105,15}$
8	105	$D4$	18	$\phi_{105,15}$	105	1^{105}		$\phi_{105,15}$
9	875	$D4(a1)$	16	$\phi_{315,16}$	315	$2^{70}3^{245}$		$\phi_{315,16} + \phi_{280,17} + \phi_{280,18}$
19	665	$D4(a1)$	16	$\phi_{315,16}$	315	$1^{35}2^{210}3^{70}$		$\phi_{315,16} + \phi_{280,17} + \phi_{70,18}$
20	875	$D4(a1)$	16	$\phi_{315,16}$	315	$2^{70}3^{245}$		$\phi_{315,16} + \phi_{280,17} + \phi_{280,18}$
16	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}		$\phi_{168,21}$
22	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}		$\phi_{168,21}$
17	210	$A3$	10	$\phi_{210,21}$	210	1^{210}		$\phi_{210,21}$
21	210	$A3$	10	$\phi_{210,21}$	210	1^{210}		$\phi_{210,21}$
23	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}		$\phi_{189,22}$
24	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}		$\phi_{189,22}$
25	225	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{15}2^{105}$		$\phi_{120,25} + \phi_{105,26}$
18	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$		$\phi_{56,30} + \phi_{35,31}$
26	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$		$\phi_{56,30} + \phi_{35,31}$
27	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$		$\phi_{56,30} + \phi_{35,31}$
28	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}		$\phi_{27,37}$
29	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}		$\phi_{27,37}$
30	7	$A1$	2	$\phi_{7,46}$	7	1^7		$\phi_{7,46}$
31	1	0	1	$\phi_{1,63}$	1	1^1		$\phi_{1,63}$

31 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
2	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
3	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
4	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
6	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
8	77	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{35}2^{21}$	$\phi_{56,3} + \phi_{21,6}$
5	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
7	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
17	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
18	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
9	225	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{15}2^{105}$	$\phi_{120,4} + \phi_{105,5}$
10	225	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{15}2^{105}$	$\phi_{120,4} + \phi_{105,5}$
12	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
13	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
11	168	$D5(a1)$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
14	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
15	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
19	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
20	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
16	756	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{84}2^{336}$	$\phi_{420,10} + \phi_{336,11}$
21	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$
22	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$

32 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W -rep
0	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
2	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
3	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
4	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
5	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
13	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
6	225	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{15}2^{105}$	$\phi_{120,4} + \phi_{105,5}$
7	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
9	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
10	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
14	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
8	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
15	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
11	875	$E7(a5)$	34	$\phi_{315,7}$	315	$2^{70}3^{245}$	$\phi_{315,7} + \phi_{280,8} + \phi_{280,9}$
12	665	$E7(a5)$	34	$\phi_{315,7}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,7} + \phi_{280,8} + \phi_{70,9}$
21	875	$E7(a5)$	34	$\phi_{315,7}$	315	$2^{70}3^{245}$	$\phi_{315,7} + \phi_{280,8} + \phi_{280,9}$
16	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
17	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
23	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
18	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$
22	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
29	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
19	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
24	756	$A4$	21	$\phi_{420,13}$	420	$1^{84}2^{336}$	$\phi_{420,13} + \phi_{336,14}$
20	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
25	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
26	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
30	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
27	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
28	135	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{105}2^{15}$	$\phi_{120,25} + \phi_{15,28}$
31	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$

33 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	$W\text{-rep}$
0	1	$E7$	45	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	7	$E7(a1)$	44	$\phi_{7,1}$	7	1^7	$\phi_{7,1}$
2	27	$E7(a2)$	43	$\phi_{27,2}$	27	1^{27}	$\phi_{27,2}$
3	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
4	91	$E7(a3)$	42	$\phi_{56,3}$	56	$1^{21}2^{35}$	$\phi_{56,3} + \phi_{35,4}$
5	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
7	21	$E6$	41	$\phi_{21,3}$	21	1^{21}	$\phi_{21,3}$
6	135	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{105}2^{15}$	$\phi_{120,4} + \phi_{15,7}$
8	135	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{105}2^{15}$	$\phi_{120,4} + \phi_{15,7}$
10	135	$E6(a1)$	40	$\phi_{120,4}$	120	$1^{105}2^{15}$	$\phi_{120,4} + \phi_{15,7}$
9	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
11	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
14	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
16	189	$E7(a4)$	38	$\phi_{189,5}$	189	1^{189}	$\phi_{189,5}$
12	210	$D6(a1)$	37	$\phi_{210,6}$	210	1^{210}	$\phi_{210,6}$
13	168	$D5 + A1$	36	$\phi_{168,6}$	168	1^{168}	$\phi_{168,6}$
15	105	$A6$	35	$\phi_{105,6}$	105	1^{105}	$\phi_{105,6}$
17	105	$A6$	35	$\phi_{105,6}$	105	1^{105}	$\phi_{105,6}$
21	105	$A6$	35	$\phi_{105,6}$	105	1^{105}	$\phi_{105,6}$
24	105	$A6$	35	$\phi_{105,6}$	105	1^{105}	$\phi_{105,6}$
18	875	$E7(a5)$	34	$\phi_{315,7}$	315	$2^{70}3^{245}$	$\phi_{315,7} + \phi_{280,8} + \phi_{280,9}$
26	665	$E7(a5)$	34	$\phi_{315,7}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,7} + \phi_{280,8} + \phi_{70,9}$
20	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
23	189	$D5$	33	$\phi_{189,7}$	189	1^{189}	$\phi_{189,7}$
22	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$	$\phi_{405,8} + \phi_{216,9}$
25	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$	$\phi_{405,8} + \phi_{216,9}$
28	621	$E6(a3)$	32	$\phi_{405,8}$	405	$1^{189}2^{216}$	$\phi_{405,8} + \phi_{216,9}$
19	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
29	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
30	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
31	378	$D5(a1) + A1$	30	$\phi_{378,9}$	378	1^{378}	$\phi_{378,9}$
33	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}	$\phi_{210,10}$
34	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}	$\phi_{210,10}$
35	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}	$\phi_{210,10}$
36	210	$A4 + A2$	27	$\phi_{210,10}$	210	1^{210}	$\phi_{210,10}$
32	504	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{336}2^{84}$	$\phi_{420,10} + \phi_{84,12}$
38	756	$D5(a1)$	26	$\phi_{420,10}$	420	$1^{84}2^{336}$	$\phi_{420,10} + \phi_{336,11}$
37	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$
39	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$
40	1024	$A4 + A1$	25	$\phi_{512,11}$	512	2^{512}	$\phi_{512,11} + \phi_{512,12}$

33 Block cont'd

#	$ C $	\mathcal{O}_C	$\#_{\mathcal{O}}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_{\infty}}$	W -rep
45	105	$(A5)''$	23	$\phi_{105,12}$	105	1^{105}	$\phi_{105,12}$
41	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
44	210	$A3 + A2 + A1$	22	$\phi_{210,13}$	210	1^{210}	$\phi_{210,13}$
46	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$	$\phi_{420,13} + \phi_{84,15}$
47	504	$A4$	21	$\phi_{420,13}$	420	$1^{336}2^{84}$	$\phi_{420,13} + \phi_{84,15}$
49	504	$A4$	21	$\phi_{420,13}$	420	$1^{84}2^{336}$	$\phi_{420,13} + \phi_{84,15}$
48	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
50	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
51	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
57	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
59	378	$A3 + A2$	20	$\phi_{378,14}$	378	1^{378}	$\phi_{378,14}$
42	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
52	621	$D4(a1) + A1$	19	$\phi_{405,15}$	405	$1^{189}2^{216}$	$\phi_{405,15} + \phi_{216,16}$
53	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$
55	105	$D4$	18	$\phi_{105,15}$	105	1^{105}	$\phi_{105,15}$
54	665	$D4(a1)$	16	$\phi_{315,16}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,16} + \phi_{280,17} + \phi_{70,18}$
56	665	$D4(a1)$	16	$\phi_{315,16}$	315	$1^{35}2^{210}3^{70}$	$\phi_{315,16} + \phi_{280,17} + \phi_{70,18}$
58	875	$D4(a1)$	16	$\phi_{315,16}$	315	$2^{70}3^{245}$	$\phi_{315,16} + \phi_{280,17} + \phi_{280,18}$
63	189	$(A3 + A1)''$	13	$\phi_{189,20}$	189	1^{189}	$\phi_{189,20}$
43	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
61	105	$A2 + 3A1$	12	$\phi_{105,21}$	105	1^{105}	$\phi_{105,21}$
64	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
65	168	$2A2$	11	$\phi_{168,21}$	168	1^{168}	$\phi_{168,21}$
66	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
69	210	$A3$	10	$\phi_{210,21}$	210	1^{210}	$\phi_{210,21}$
62	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
67	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
68	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
70	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
71	189	$A2 + 2A1$	9	$\phi_{189,22}$	189	1^{189}	$\phi_{189,22}$
72	135	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{105}2^{15}$	$\phi_{120,25} + \phi_{15,28}$
75	225	$A2 + A1$	8	$\phi_{120,25}$	120	$1^{15}2^{105}$	$\phi_{120,25} + \phi_{105,26}$
73	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
74	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
76	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
77	91	$A2$	6	$\phi_{56,30}$	56	$1^{21}2^{35}$	$\phi_{56,30} + \phi_{35,31}$
78	21	$(3A1)''$	4	$\phi_{21,36}$	21	1^{21}	$\phi_{21,36}$
79	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
80	27	$2A1$	3	$\phi_{27,37}$	27	1^{27}	$\phi_{27,37}$
81	7	$A1$	2	$\phi_{7,46}$	7	1^7	$\phi_{7,46}$
82	7	$A1$	2	$\phi_{7,46}$	7	1^7	$\phi_{7,46}$
83	1	0	1	$\phi_{1,63}$	1	1^1	$\phi_{1,63}$
84	1	0	1	$\phi_{1,63}$	1	1^1	$\phi_{1,63}$

7. E_8

11 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_{\infty}}$	W -rep
0	525	E_6	50	$\phi_{525,12}$	525	1^{525}	$\phi_{525,12}$
1	2100	D_5	34	$\phi_{2100,20}$	2100	1^{2100}	$\phi_{2100,20}$
2	525	D_4	15	$\phi_{525,36}$	525	1^{525}	$\phi_{525,36}$

12 Block

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W-rep
0	525	E_6	50	$\phi_{525,12}$	525	1^{525}	$\phi_{525,12}$
1	3500	$E_6(a_1)$	47	$\phi_{2800,13}$	2800	$1^{2100}2^{700}$	$\phi_{2800,13} + \phi_{700,16}$
5	4200	A_6	43	$\phi_{4200,15}$	4200	1^{4200}	$\phi_{4200,15}$
2	2100	D_5	34	$\phi_{2100,20}$	2100	1^{2100}	$\phi_{2100,20}$
3	2100	D_5	34	$\phi_{2100,20}$	2100	1^{2100}	$\phi_{2100,20}$
4	8800	$E_6(a_3)$	33	$\phi_{5600,21}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,21} + \phi_{3200,22}$
6	8800	$E_6(a_3)$	33	$\phi_{5600,21}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,21} + \phi_{3200,22}$
7	4536	$A_4 + A_2$	28	$\phi_{4536,23}$	4536	1^{4536}	$\phi_{4536,23}$
8	4900	$D_5(a_1)$	26	$\phi_{2800,25}$	2800	$1^{700}2^{2100}$	$\phi_{2800,25} + \phi_{2100,28}$
9	8192	$A_4 + A_1$	24	$\phi_{4096,26}$	4096	2^{4096}	$\phi_{4096,26} + \phi_{4096,27}$
11	3240	A_4	20	$\phi_{2268,30}$	2268	$1^{1296}2^{972}$	$\phi_{2268,30} + \phi_{972,32}$
14	3240	A_4	20	$\phi_{2268,30}$	2268	$1^{1296}2^{972}$	$\phi_{2268,30} + \phi_{972,32}$
18	3240	$A_3 + A_2$	19	$\phi_{3240,31}$	3240	1^{3240}	$\phi_{3240,31}$
10	525	D_4	15	$\phi_{525,36}$	525	1^{525}	$\phi_{525,36}$
13	525	D_4	15	$\phi_{525,36}$	525	1^{525}	$\phi_{525,36}$
12	3192	$D_4(a_1)$	14	$\phi_{1400,37}$	1400	$1^{56}2^{896}3^{448}$	$\phi_{1400,37} + \phi_{1344,38} + \phi_{448,39}$
15	3752	$D_4(a_1)$	14	$\phi_{1400,37}$	1400	$2^{448}3^{952}$	$\phi_{1400,37} + \phi_{1344,38} + \phi_{1008,39}$
19	3752	$D_4(a_1)$	14	$\phi_{1400,37}$	1400	$2^{448}3^{952}$	$\phi_{1400,37} + \phi_{1344,38} + \phi_{1008,39}$
20	1000	$2A_2$	11	$\phi_{700,42}$	700	$1^{400}2^{300}$	$\phi_{700,42} + \phi_{300,44}$
16	567	A_3	9	$\phi_{567,46}$	567	1^{567}	$\phi_{567,46}$
21	567	A_3	9	$\phi_{567,46}$	567	1^{567}	$\phi_{567,46}$
22	560	$A_2 + 2A_1$	8	$\phi_{560,47}$	560	1^{560}	$\phi_{560,47}$
24	560	$A_2 + 2A_1$	8	$\phi_{560,47}$	560	1^{560}	$\phi_{560,47}$
25	370	$A_2 + A_1$	7	$\phi_{210,52}$	210	$1^{50}2^{160}$	$\phi_{210,52} + \phi_{160,55}$
17	196	A_2	5	$\phi_{112,63}$	112	$1^{28}2^{84}$	$\phi_{112,63} + \phi_{84,64}$
23	196	A_2	5	$\phi_{112,63}$	112	$1^{28}2^{84}$	$\phi_{112,63} + \phi_{84,64}$
26	196	A_2	5	$\phi_{112,63}$	112	$1^{28}2^{84}$	$\phi_{112,63} + \phi_{84,64}$
27	35	$2A_1$	3	$\phi_{35,74}$	35	1^{35}	$\phi_{35,74}$
28	35	$2A_1$	3	$\phi_{35,74}$	35	1^{35}	$\phi_{35,74}$
29	8	A_1	2	$\phi_{8,91}$	8	1^8	$\phi_{8,91}$
30	1	1	1	$\phi_{1,120}$	1	1^1	$\phi_{1,120}$

21 Block

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	$W\text{-rep}$
0	1	E_8	70	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	8	$E_8(a_1)$	69	$\phi_{8,1}$	8	1^8	$\phi_{8,1}$
2	35	$E_8(a_2)$	68	$\phi_{35,2}$	35	1^{35}	$\phi_{35,2}$
3	35	$E_8(a_2)$	68	$\phi_{35,2}$	35	1^{35}	$\phi_{35,2}$
4	196	$E_8(a_3)$	67	$\phi_{112,3}$	112	$1^{28}2^{84}$	$\phi_{112,3} + \phi_{84,4}$
5	196	$E_8(a_3)$	67	$\phi_{112,3}$	112	$1^{28}2^{84}$	$\phi_{112,3} + \phi_{84,4}$
13	196	$E_8(a_3)$	67	$\phi_{112,3}$	112	$1^{28}2^{84}$	$\phi_{112,3} + \phi_{84,4}$
6	370	$E_8(a_4)$	66	$\phi_{210,4}$	210	$1^{50}2^{160}$	$\phi_{210,4} + \phi_{160,7}$
7	560	$E_8(b_4)$	64	$\phi_{560,5}$	560	1^{560}	$\phi_{560,5}$
9	560	$E_8(b_4)$	64	$\phi_{560,5}$	560	1^{560}	$\phi_{560,5}$
8	1000	$E_8(a_5)$	63	$\phi_{700,6}$	700	$1^{400}2^{300}$	$\phi_{700,6} + \phi_{300,8}$
10	567	$E_7(a_1)$	62	$\phi_{567,6}$	567	1^{567}	$\phi_{567,6}$
14	567	$E_7(a_1)$	62	$\phi_{567,6}$	567	1^{567}	$\phi_{567,6}$
11	3752	$E_8(b_5)$	61	$\phi_{1400,7}$	1400	$2^{448}3^{952}$	$\phi_{1400,7} + \phi_{1344,8} + \phi_{1008,9}$
12	3192	$E_8(b_5)$	61	$\phi_{1400,7}$	1400	$1^{56}2^{896}3^{448}$	$\phi_{1400,7} + \phi_{1344,8} + \phi_{448,9}$
20	3752	$E_8(b_5)$	61	$\phi_{1400,7}$	1400	$2^{448}3^{952}$	$\phi_{1400,7} + \phi_{1344,8} + \phi_{1008,9}$
15	3240	$D_7(a_1)$	56	$\phi_{3240,9}$	3240	1^{3240}	$\phi_{3240,9}$
16	3240	$E_7(a_3)$	54	$\phi_{2268,10}$	2268	$1^{1296}2^{972}$	$\phi_{2268,10} + \phi_{972,12}$
22	3240	$E_7(a_3)$	54	$\phi_{2268,10}$	2268	$1^{1296}2^{972}$	$\phi_{2268,10} + \phi_{972,12}$
17	8192	$E_6(a_1) + A_1$	53	$\phi_{4096,11}$	4096	2^{4096}	$\phi_{4096,11} + \phi_{4096,12}$
21	525	E_6	50	$\phi_{525,12}$	525	1^{525}	$\phi_{525,12}$
28	525	$A_4 + A_2$	50	$\phi_{525,12}$	525	1^{525}	$\phi_{525,12}$
18	4536	$D_5 + A_2$	48	$\phi_{4536,13}$	4536	1^{4536}	$\phi_{4536,13}$
23	4900	$E_6(a_1)$	47	$\phi_{2800,13}$	2800	$1^{700}2^{2100}$	$\phi_{2800,13} + \phi_{2100,16}$
19	8800	$D_6(a_1)$	44	$\phi_{5600,15}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,15} + \phi_{3200,16}$
24	8800	$D_6(a_1)$	44	$\phi_{5600,15}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,15} + \phi_{3200,16}$
25	2100	D_4	34	$\phi_{2100,20}$	2100	1^{2100}	$\phi_{2100,20}$
29	2100	D_5	34	$\phi_{2100,20}$	2100	1^{2100}	$\phi_{2100,20}$
26	4200	$D_4 + A_2$	32	$\phi_{4200,21}$	4200	1^{4200}	$\phi_{4200,21}$
27	3500	$D_5(a_1)$	26	$\phi_{2800,25}$	2800	$1^{2100}2^{700}$	$\phi_{2800,25} + \phi_{700,28}$
30	525	D_4	15	$\phi_{525,36}$	525	1^{525}	$\phi_{525,36}$

22 Block

#	C	\mathcal{O}_C	$\# \mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W-rep?
0	1	E_8	70	$\phi_{1,0}$	1	1^1	$\phi_{1,0}$
1	8	$E_8(a_1)$	69	$\phi_{8,1}$	8	1^8	$\phi_{8,1}$
2	35	$E_8(a_2)$	68	$\phi_{35,2}$	35	1^{35}	$\phi_{35,2}$
3	196	$E_8(a_3)$	67	$\phi_{112,3}$	112	$1^{28}2^{84}$	$\phi_{112,3} + \phi_{84,4}$
5	196	$E_8(a_3)$	67	$\phi_{112,3}$	112	$1^{28}2^{84}$	$\phi_{112,3} + \phi_{84,4}$
4	260	$E_8(a_4)$	66	$\phi_{210,4}$	210	$1^{160}2^{50}$	$\phi_{210,4} + \phi_{50,8}$
7	260	$E_8(a_4)$	66	$\phi_{210,4}$	210	$1^{160}2^{50}$	$\phi_{210,4} + \phi_{50,8}$
6	560	$E_8(b_4)$	64	$\phi_{560,5}$	560	1^{560}	$\phi_{560,5}$
8	560	$E_8(b_4)$	64	$\phi_{560,5}$	560	1^{560}	$\phi_{560,5}$
9	560	$E_8(b_4)$	64	$\phi_{560,5}$	560	1^{560}	$\phi_{560,5}$
10	1100	$E_8(a_5)$	63	$\phi_{700,6}$	700	$1^{300}2^{400}$	$\phi_{700,6} + \phi_{400,7}$
13	1100	$E_8(a_5)$	63	$\phi_{700,6}$	700	$1^{300}2^{400}$	$\phi_{700,6} + \phi_{400,7}$
17	1100	$E_8(a_5)$	63	$\phi_{700,6}$	700	$1^{300}2^{400}$	$\phi_{700,6} + \phi_{400,7}$
11	567	$E_7(a_1)$	62	$\phi_{567,6}$	567	1^{567}	$\phi_{567,6}$
12	3192	$E_8(b_5)$	61	$\phi_{1400,7}$	1400	$1^{56}2^{896}3^{448}$	$\phi_{1400,7} + \phi_{1344,8} + \phi_{448,9}$
14	3752	$E_8(b_5)$	61	$\phi_{1400,7}$	1400	$2^{448}3^{952}$	$\phi_{1400,7} + \phi_{1344,8} + \phi_{1008,9}$
15	4025	$E_8(a_6)$	59	$\phi_{1400,8}$	1400	$2^{175}3^{1225}$	$\phi_{1400,8} + \phi_{1575,10} + \phi_{1050,10}$
18	2625	$E_8(a_6)$	59	$\phi_{1400,8}$	1400	$1^{350}2^{875}3^{175}$	$\phi_{1400,8} + \phi_{1050,10} + \phi_{175,12}$
16	3240	$D_7(a_1)$	56	$\phi_{3240,9}$	3240	1^{3240}	$\phi_{3240,9}$
19	3240	$D_7(a_1)$	56	$\phi_{3240,9}$	3240	1^{3240}	$\phi_{3240,9}$
20	3240	$D_7(a_1)$	56	$\phi_{3240,9}$	3240	1^{3240}	$\phi_{3240,9}$
21	3240	$D_7(a_1)$	56	$\phi_{3240,9}$	3240	1^{3240}	$\phi_{3240,9}$
22	3240	$D_7(a_1)$	56	$\phi_{3240,9}$	3240	1^{3240}	$\phi_{3240,9}$
23	3640	$E_8(b_6)$	55	$\phi_{2240,10}$	2240	$1^{840}2^{1400}$	$\phi_{2240,10} + \phi_{1400,11}$
25	3640	$E_8(b_6)$	55	$\phi_{2240,10}$	2240	$1^{840}2^{1400}$	$\phi_{2240,10} + \phi_{1400,11}$
27	3640	$E_8(b_6)$	55	$\phi_{2240,10}$	2240	$1^{840}2^{1400}$	$\phi_{2240,10} + \phi_{1400,11}$
24	3240	$E_7(a_3)$	54	$\phi_{2268,10}$	2268	$1^{1296}2^{972}$	$\phi_{2268,10} + \phi_{972,12}$
29	3240	$E_7(a_3)$	54	$\phi_{2268,10}$	2268	$1^{1296}2^{972}$	$\phi_{2268,10} + \phi_{972,12}$
26	8192	$E_6(a_1) + A_1$	53	$\phi_{4096,11}$	4096	2^{4096}	$\phi_{4096,11} + \phi_{4096,12}$
30	8192	$E_6(a_1) + A_1$	53	$\phi_{4096,11}$	4096	2^{4096}	$\phi_{4096,11} + \phi_{4096,12}$
28	7560	$D_7(a_2)$	51	$\phi_{4200,12}$	4200	$1^{840}2^{3360}$	$\phi_{4200,12} + \phi_{3360,13}$
31	5040	$D_7(a_2)$	51	$\phi_{4200,12}$	4200	$1^{3360}2^{840}$	$\phi_{4200,12} + \phi_{840,14}$
35	7560	$D_7(a_2)$	51	$\phi_{4200,12}$	4200	$1^{840}2^{3360}$	$\phi_{4200,12} + \phi_{3360,13}$
33	525	E_6	50	$\phi_{525,12}$	525	1^{525}	$\phi_{525,12}$
32	4536	$D_5 + A_2$	48	$\phi_{4536,13}$	4536	1^{4536}	$\phi_{4536,13}$
38	4536	$D_5 + A_2$	48	$\phi_{4536,13}$	4536	1^{4536}	$\phi_{4536,13}$
39	4536	$D_5 + A_2$	48	$\phi_{4536,13}$	4536	1^{4536}	$\phi_{4536,13}$
45	4536	$D_5 + A_2$	48	$\phi_{4536,13}$	4536	1^{4536}	$\phi_{4536,13}$
34	3500	$E_6(a_1)$	47	$\phi_{2800,13}$	2800	$1^{2100}2^{700}$	$\phi_{2800,13} + \phi_{700,16}$
37	3500	$E_6(a_1)$	47	$\phi_{2800,13}$	2800	$1^{2100}2^{700}$	$\phi_{2800,13} + \phi_{700,16}$
36	6075	$E_7(a_4)$	46	$\phi_{6075,14}$	6075	1^{6075}	$\phi_{6075,14}$
41	6075	$E_7(a_4)$	46	$\phi_{6075,14}$	6075	1^{6075}	$\phi_{6075,14}$
43	6075	$E_7(a_4)$	46	$\phi_{6075,14}$	6075	1^{6075}	$\phi_{6075,14}$
40	2835	$A_6 + A_1$	45	$\phi_{2835,14}$	2835	1^{2835}	$\phi_{2835,14}$
44	8800	$D_6(a_1)$	44	$\phi_{5600,15}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,15} + \phi_{3200,16}$
49	8800	$D_6(a_1)$	44	$\phi_{5600,15}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,15} + \phi_{3200,16}$
42	4200	A_6	43	$\phi_{4200,15}$	4200	1^{4200}	$\phi_{4200,15}$
46	4200	A_6	43	$\phi_{4200,15}$	4200	1^{4200}	$\phi_{4200,15}$
48	4200	A_6	43	$\phi_{4200,15}$	4200	1^{4200}	$\phi_{4200,15}$
50	4200	A_6	43	$\phi_{4200,15}$	4200	1^{4200}	$\phi_{4200,15}$
47	22778	$E_8(a_7)$	42	$\phi_{4480,16}$	4480	$2^{70}3^{378}4^{1092}$ $5^{924}6^{1596}7^{420}$	$\phi_{4480,16} + \phi_{7168,17} + \phi_{3150,18}$ $+ \phi_{4200,18} + \phi_{1344,19} + \phi_{2016,19}$ $+ \phi_{420,20}$
51	38766	$E_8(a_7)$	42	$\phi_{4480,16}$	4480	$5^{420}7^{1134}$ $8^{1596}12^{1330}$	$\phi_{4480,16} + \phi_{7168,17} + 2\phi_{4200,18}$ $+ \phi_{4536,18} + \phi_{5670,18} + \phi_{1344,19}$ $+ \phi_{5600,19} + \phi_{1400,20} + \phi_{168,24}$
53	46676	$E_8(a_7)$	42	$\phi_{4480,16}$	4480	$6^{420}7^{756}8^{168}$ $9^{378}11^{70}$ $12^{1596}13^{1092}$	$\phi_{4480,16} + 2\phi_{7168,17} + \phi_{3150,18}$ $+ \phi_{4200,18} + \phi_{4536,18} + \phi_{5670,18}$ $+ \phi_{2016,19} + \phi_{5600,19} + \phi_{2688,20}$

#	$ C $	\mathcal{O}_C	$\#o$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	W-rep?
54	2100	D_5	34	$\phi_{2100,20}$	2100	1^{2100}	$\phi_{2100,20}$
55	8800	$E_6(a_3)$	33	$\phi_{5600,21}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,21} + \phi_{3200,22}$
63	8800	$E_6(a_3)$	33	$\phi_{5600,21}$	5600	$1^{2400}2^{3200}$	$\phi_{5600,21} + \phi_{3200,22}$
52	4200	$D_4 + A_2$	32	$\phi_{4200,21}$	4200	1^{4200}	$\phi_{4200,21}$
57	4200	$D_4 + A_2$	32	$\phi_{4200,21}$	4200	1^{4200}	$\phi_{4200,21}$
59	4200	$D_4 + A_2$	32	$\phi_{4200,21}$	4200	1^{4200}	$\phi_{4200,21}$
62	4200	$D_4 + A_2$	32	$\phi_{4200,21}$	4200	1^{4200}	$\phi_{4200,21}$
60	2835	$A_4 + A_2 + A_1$	31	$\phi_{2835,22}$	2835	1^{2835}	$\phi_{2835,22}$
58	6075	$D_5(a_1) + A_1$	30	$\phi_{6075,22}$	6075	1^{6075}	$\phi_{6075,22}$
64	6075	$D_5(a_1) + A_1$	30	$\phi_{6075,22}$	6075	1^{6075}	$\phi_{6075,22}$
65	6075	$D_5(a_1) + A_1$	30	$\phi_{6075,22}$	6075	1^{6075}	$\phi_{6075,22}$
56	4536	$A_4 + A_2$	28	$\phi_{4536,23}$	4536	1^{4536}	$\phi_{4536,23}$
66	4536	$A_4 + A_2$	28	$\phi_{4536,23}$	4536	1^{4536}	$\phi_{4536,23}$
68	4536	$A_4 + A_2$	28	$\phi_{4536,23}$	4536	1^{4536}	$\phi_{4536,23}$
69	4536	$A_4 + A_2$	28	$\phi_{4536,23}$	4536	1^{4536}	$\phi_{4536,23}$
61	7560	$A_4 + 2A_1$	27	$\phi_{4200,24}$	4200	$1^{840}2^{3360}$	$\phi_{4200,24} + \phi_{3360,25}$
67	7560	$A_4 + 2A_1$	27	$\phi_{4200,24}$	4200	$1^{840}2^{3360}$	$\phi_{4200,24} + \phi_{3360,25}$
70	5040	$A_4 + 2A_1$	27	$\phi_{4200,24}$	4200	$1^{3360}2^{840}$	$\phi_{4200,24} + \phi_{840,26}$
71	3500	$D_5(a_1)$	26	$\phi_{2800,25}$	2800	$1^{2100}2^{700}$	$\phi_{2800,25} + \phi_{700,28}$
79	3500	$D_5(a_1)$	26	$\phi_{2800,25}$	2800	$1^{2100}2^{700}$	$\phi_{2800,25} + \phi_{700,28}$
72	8192	$A_4 + A_1$	24	$\phi_{4096,26}$	4096	2^{4096}	$\phi_{4096,26} + \phi_{4096,27}$
80	8192	$A_4 + A_1$	24	$\phi_{4096,26}$	4096	2^{4096}	$\phi_{4096,26} + \phi_{4096,27}$
73	3640	$D_4(a_1) + A_2$	23	$\phi_{2240,28}$	2240	$1^{840}2^{1400}$	$\phi_{2240,28} + \phi_{1400,29}$
78	3640	$D_4(a_1) + A_2$	23	$\phi_{2240,28}$	2240	$1^{840}2^{1400}$	$\phi_{2240,28} + \phi_{1400,29}$
81	3640	$D_4(a_1) + A_2$	23	$\phi_{2240,28}$	2240	$1^{840}2^{1400}$	$\phi_{2240,28} + \phi_{1400,29}$
74	3240	A_4	20	$\phi_{2268,30}$	2268	$1^{1296}2^{972}$	$\phi_{2268,30} + \phi_{972,32}$
82	3240	A_4	20	$\phi_{2268,30}$	2268	$1^{1296}2^{972}$	$\phi_{2268,30} + \phi_{972,32}$
75	3240	$A_3 + A_2$	19	$\phi_{3240,31}$	3240	1^{3240}	$\phi_{3240,31}$
76	3240	$A_3 + A_2$	19	$\phi_{3240,31}$	3240	1^{3240}	$\phi_{3240,31}$
83	3240	$A_3 + A_2$	19	$\phi_{3240,31}$	3240	1^{3240}	$\phi_{3240,31}$
84	3240	$A_3 + A_2$	19	$\phi_{3240,31}$	3240	1^{3240}	$\phi_{3240,31}$
86	3240	$A_3 + A_2$	19	$\phi_{3240,31}$	3240	1^{3240}	$\phi_{3240,31}$
87	4025	$D_4(a_1) + A_1$	18	$\phi_{1400,32}$	1400	$2^{175}3^{1225}$	$\phi_{1400,32} + \phi_{1575,34} + \phi_{1050,34}$
89	2625	$D_4(a_1) + A_1$	18	$\phi_{1400,32}$	1400	$1^{350}2^{875}3^{175}$	$\phi_{1400,32} + \phi_{1050,34} + \phi_{175,36}$
85	525	D_4	15	$\phi_{525,36}$	525	1^{525}	$\phi_{525,36}$
88	3752	$D_4(a_1)$	14	$\phi_{1400,37}$	1400	$2^{448}3^{952}$	$\phi_{1400,37} + \phi_{1344,38} + \phi_{1008,39}$
90	3192	$D_4(a_1)$	14	$\phi_{1400,37}$	1400	$1^{56}2^{896}3^{448}$	$\phi_{1400,37} + \phi_{1344,38} + \phi_{448,39}$
77	1100	$2A_2$	11	$\phi_{700,42}$	700	$1^{300}2^{400}$	$\phi_{700,42} + \phi_{400,43}$
91	1100	$2A_2$	11	$\phi_{700,42}$	700	$1^{300}2^{400}$	$\phi_{700,42} + \phi_{400,43}$
92	1100	$2A_2$	11	$\phi_{700,42}$	700	$1^{300}2^{400}$	$\phi_{700,42} + \phi_{400,43}$
94	567	A_3	9	$\phi_{567,46}$	567	1^{567}	$\phi_{567,46}$
93	560	$A_2 + 2A_1$	8	$\phi_{560,47}$	560	1^{560}	$\phi_{560,47}$
95	560	$A_2 + 2A_1$	8	$\phi_{560,47}$	560	1^{560}	$\phi_{560,47}$
96	560	$A_2 + 2A_1$	8	$\phi_{560,47}$	560	1^{560}	$\phi_{560,47}$
97	260	$A_2 + A_1$	7	$\phi_{210,52}$	210	$1^{160}2^{50}$	$\phi_{210,52} + \phi_{50,56}$
99	260	$A_2 + A_1$	7	$\phi_{210,52}$	210	$1^{160}2^{50}$	$\phi_{210,52} + \phi_{50,56}$
98	196	A_2	5	$\phi_{112,63}$	112	$1^{28}2^{84}$	$\phi_{112,63} + \phi_{84,64}$
100	196	A_2	5	$\phi_{112,63}$	112	$1^{28}2^{84}$	$\phi_{112,63} + \phi_{84,64}$
101	35	$2A_1$	3	$\phi_{35,74}$	35	1^{35}	$\phi_{35,74}$
102	8	A_1	2	$\phi_{8,91}$	8	1^8	$\phi_{8,91}$
103	1	0	1	$\phi_{1,120}$	1	1^1	$\phi_{1,120}$

8. $G_2(\mathbb{C})$

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	$W\text{-rep?}$
0	1	G_2	5	$\phi_{1,0}$	1	1^1	$(\phi_{1,0})^2$
2	10	$G_2(a_1)$	4	$\phi_{2,1}$	4	$2^2 3^2$	$(\phi_{2,1})^2 + (\phi_{2,2})^2 + (\phi'_{1,3})^2 + (\phi''_{1,3})^2$
3	1	0	1	$\phi_{1,6}$	1	1^1	$(\phi_{1,6})^2$

9. $F_4(\mathbb{C})$

#	$ C $	\mathcal{O}_C	n	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	$W\text{-rep?}$
0	1	F_4	16	$\phi_{1,0}$	1	1^1	$(\phi_{1,0})^2$
1	24	$F_4(a_1)$	15	$\phi_{4,1}$	16	$1^8 2^8$	$(\phi_{4,1})^2 + (\phi'_{2,4})^2 + (\phi''_{2,4})^2$
2	81	$F_4(a_2)$	14	$\phi_{9,2}$	81	1^{81}	$(\phi_{9,2})^2$
3	64	B_3	12	$\phi''_{8,3}$	64	1^{64}	$(\phi''_{8,3})^2$
4	64	C_3	13	$\phi'_{8,3}$	64	1^{64}	$(\phi'_{8,3})^2$
5	684	$F_4(a_3)$	11	$\phi_{12,4}$	144	$2^{12} 3^{20} 4^{48} 5^{30} 6^{16} 9^{18}$	$(\phi_{12,4})^2 + (\phi_{16,5})^2 + (\phi'_{6,6})^2 + (\phi''_{6,6})^2 + (\phi'_{9,6})^2 + (\phi''_{9,6})^2 + (\phi_{4,7})^2 + (\phi'_{4,7})^2 + (\phi_{4,8})^2 + (\phi'_{1,12})^2 + (\phi''_{1,12})^2$
6	64	A_2	5	$\phi''_{8,9}$	64	1^{64}	$(\phi''_{8,9})^2$
7	64	\tilde{A}_2	6	$\phi'_{8,9}$	64	1^{64}	$(\phi'_{8,9})^2$
8	81	$A_1 + \tilde{A}_1$	4	$\phi_{9,10}$	81	1^{81}	$(\phi_{9,10})^2$
9	24	\tilde{A}_1	3	$\phi_{4,13}$	16	$1^8 2^8$	$(\phi_{4,13})^2 + (\phi'_{2,16})^2 + (\phi''_{2,16})^2$
10	1	0	1	$\phi_{1,24}$	1	1^1	$(\phi_{1,24})^2$

10. $E_6(\mathbb{C})$

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	$W\text{-rep?}$
0	1	E_6	21	$\phi_{1,0}$	1	1^1	$(\phi_{1,0})^2$
1	36	$E_6(a_1)$	20	$\phi_{6,1}$	36	1^{36}	$(\phi_{6,1})^2$
2	400	D_5	19	$\phi_{20,2}$	400	1^{400}	$(\phi_{20,2})^2$
3	1350	$E_6(a_3)$	18	$\phi_{30,3}$	900	$1^{450} 2^{450}$	$(\phi_{30,3})^2 + (\phi_{15,4})^2 + (\phi_{15,5})^2$
4	4096	$D_5(a_1)$	17	$\phi_{64,4}$	4096	1^{4096}	$(\phi_{64,4})^2$
5	3600	$A_4 + A_1$	15	$\phi_{60,5}$	3600	1^{3600}	$(\phi_{60,5})^2$
6	6561	A_4	13	$\phi_{81,6}$	6561	1^{6561}	$(\phi_{81,6})^2$
7	576	D_4	13	$\phi_{24,6}$	576	1^{576}	$(\phi_{24,6})^2$
8	18600	$D_4(a_1)$	12	$\phi_{80,7}$	6400	$1^{400} 2^{1000} 3^{4600} 6^{400}$	$(\phi_{80,7})^2 + (\phi_{90,8})^2 + (\phi_{60,8})^2 + (\phi_{10,9})^2 + (\phi_{20,10})^2$
9	6561	$A_3 + A_1$	9	$\phi_{81,10}$	6561	1^{6561}	$(\phi_{81,10})^2$
10	3600	$A_2 + 2A_1$	8	$\phi_{60,11}$	3600	1^{3600}	$(\phi_{60,11})^2$
11	576	$2A_2$	7	$\phi_{24,12}$	576	1^{576}	$(\phi_{24,12})^2$
12	4096	$A_2 + A_1$	6	$\phi_{64,13}$	4096	1^{4096}	$(\phi_{64,13})^2$
13	1350	A_2	5	$\phi_{30,15}$	900	$1^{450} 2^{450}$	$(\phi_{30,15})^2 + (\phi_{15,16})^2 + (\phi_{15,17})^2$
14	400	$2A_1$	3	$\phi_{20,20}$	400	1^{400}	$(\phi_{20,20})^2$
15	36	A_1	2	$\phi_{6,25}$	36	1^{36}	$(\phi_{6,25})^2$
16	1	0	1	$\phi_{1,36}$	1	1^1	$(\phi_{1,36})^2$

11. $E_7(\mathbb{C})$

#	$ C $	\mathcal{O}_C	$\#\mathcal{O}$	σ_{spec}	$\#Prim(C)$	$\mathcal{P}_{\tau_\infty}$	$W\text{-rep?}$
0	1	E_7	45	$\phi_{1,0}$	1	1^1	$(\phi_{1,0})^2$
1	49	$E_7(a_1)$	44	$\phi_{7,1}$	49	1^{49}	$(\phi_{7,1})^2$
2	729	$E_7(a_2)$	43	$\phi_{27,2}$	729	1^{729}	$(\phi_{27,2})^2$
3	4802	$E_7(a_3)$	42	$\phi_{56,3}$	3136	$1^{1470}2^{1666}$	$(\phi_{56,3})^2 + (\phi_{35,4})^2 + (\phi_{21,6})^2$
4	441	E_6	41	$\phi_{21,3}$	441	1^{441}	$(\phi_{21,3})^2$
5	25650	$E_6(a_1)$	40	$\phi_{120,4}$	14400	$1^{3150}2^{11250}$	$(\phi_{120,4})^2 + (\phi_{105,5})^2 + (\phi_{15,7})^2$
6	35721	$E_7(a_4)$	38	$\phi_{189,5}$	35721	1^{35721}	$(\phi_{189,5})^2$
7	44100	$D_6(a_1)$	37	$\phi_{210,6}$	44100	1^{44100}	$(\phi_{210,6})^2$
8	28224	$D_5 + A_1$	36	$\phi_{168,6}$	28224	1^{28224}	$(\phi_{168,6})^2$
9	11025	A_6	35	$\phi_{105,6}$	11025	1^{11025}	$(\phi_{105,6})^2$
10	262150	$E_7(a_5)$	34	$\phi_{315,7}$	99225	$1^{4900}2^{29400}3^{63700}6^{1225}$	$(\phi_{315,7})^2 + (\phi_{280,8})^2 + (\phi_{280,9})^2 + (\phi_{35,13})^2$
11	35721	D_5	33	$\phi_{189,7}$	35721	1^{35721}	$(\phi_{189,7})^2$
12	246402	$E_6(a_3)$	32	$\phi_{405,8}$	164025	$1^{81648}2^{82377}$	$(\phi_{405,8})^2 + (\phi_{216,9})^2 + (\phi_{189,10})^2$
13	142884	$D_5(a_1) + A_1$	30	$\phi_{378,9}$	142884	1^{142884}	$(\phi_{378,9})^2$
14	296352	$D_5(a_1)$	26	$\phi_{420,10}$	176400	$1^{56448}2^{119952}$	$(\phi_{210,10})^2 + (\phi_{336,11})^2 + (\phi_{84,12})^2$
15	44100	$A_4 + A_2$	26	$\phi_{210,10}$	44100	1^{44100}	$(\phi_{210,10})^2$
16	524288	$A_4 + A_1$	25	$\phi_{512,11}$	262144	1^{262144}	$(\phi_{512,11})^2$
17	11025	$(A_5)''$	23	$\phi_{105,12}$	11025	1^{11025}	$(\phi_{105,12})^2$
18	296352	A_4	21	$\phi_{420,13}$	176400	$1^{56448}2^{119952}$	$(\phi_{420,13})^2 + (\phi_{336,14})^2 + (\phi_{84,15})^2$
19	44100	$A_3 + A_2 + A_1$	22	$\phi_{210,13}$	44100	1^{44100}	$(\phi_{210,13})^2$
20	142884	$A_3 + A_2$	20	$\phi_{378,14}$	142884	1^{142884}	$(\phi_{378,14})^2$
21	246402	$D_4(a_1) + A_1$	19	$\phi_{405,15}$	164025	$1^{81648}2^{82377}$	$(\phi_{405,15})^2 + (\phi_{216,16})^2 + (\phi_{189,17})^2$
22	11025	D_4	18	$\phi_{105,15}$	11025	1^{11025}	$(\phi_{105,15})^2$
23	262150	$D_4(a_1)$	16	$\phi_{315,16}$	99225	$1^{4900}2^{29400}3^{63700}6^{1225}$	$(\phi_{315,16})^2 + (\phi_{280,17})^2 + (\phi_{280,18})^2 + (\phi_{70,,18})^2 + (\phi_{35,22})^2$
24	35721	$(A_3 + A_1)''$	13	$\phi_{189,20}$	35721	1^{35721}	$(\phi_{189,20})^2$
25	11025	$A_2 + 3A_1$	12	$\phi_{105,21}$	11025	1^{11025}	$(\phi_{105,21})^2$
26	28224	$2A_2$	11	$\phi_{168,21}$	28224	1^{28224}	$(\phi_{168,21})^2$
27	44199	A_3	10	$\phi_{210,21}$	44100	1^{44100}	$(\phi_{210,21})^2$
28	35721	$A_2 + 2A_1$	9	$\phi_{189,22}$	35721	1^{35721}	$(\phi_{189,22})^2$
29	25650	$A_2 + A_1$	8	$\phi_{120,25}$	14400	1^{14400}	$(\phi_{120,25})^2$
30	4802	A_2	6	$\phi_{56,30}$	3136	1^{3136}	$(\phi_{56,30})^2$
31	441	$(3A_1)''$	4	$\phi_{21,36}$	441	1^{441}	$(\phi_{21,36})^2$
32	729	$2A_1$	3	$\phi_{27,37}$	721	1^{729}	$(\phi_{27,37})^2$
33	49	A_1	2	$\phi_{7,46}$	49	1^{49}	$(\phi_{7,46})^2$
34	1	0	1	$\phi_{1,63}$	1	1^1	$(\phi_{1,63})^2$