MATH 1483 **Sample Final Exam** Spring 2003

1. A salesman has fixed monthly costs and earns income by selling large-screen TVs. His profit is the same for each TV sold. Here are his recent monthly sales and net profit (income less fixed costs):

| $T$=number of TVs sold: | 16 | 25 |
|---|---|---|
| $P$ = Net Profit: | \$1280 | \$7040 |

   (a) Explain why $P$ is a linear function of $T$.

   (b) Calculate the slope of $P$ as a function of $T$ and explain what it means in terms of the TVs.

   (c) Calculate the vertical intercept of $P$ as a linear function of $T$ and explain what it means in terms of the income and expenses of the salesman.

   (d) Graph $P$ as a function of $T$ for values of $T$ from 0 to 30 TVs sold.

   (e) Calculate the horizontal intercept of $P$ as a linear function of $T$ and explain what it means in terms of the salesman's income and sales.

2. A population $N = N(t)$ is growing in accordance with the following table

| year $t$ | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| population $N$ | 1000 | 1200 | 1440 | 1728 |

   where time $t$ is measured in years.

   (a) Explain whether the data is precisely exponential or is simply close to exponential.

   (b) Find the exponential function giving the population $N$ as a function of $t$.

   (c) What is the annual percentage growth rate for $N$?

   (d) Find the decade growth factor for $N$.

   (e) What is the decade percentage growth rate of $N$?

   (f) What do you predict, based on the formula in (b), the population to be in the year $t = 5$?

3. Here is a data table showing how fast various animals run as a function of length:

| Animal | Length $L$ | Speed $R$ |
|---|---|---|
| Deermouse | 3.5 | 8.2 |
| Chipmunk | 6.3 | 15.7 |
| Red Fox | 24 | 65.6 |
| Cheetah | 47 | 95.1 |

Here length $L$ is in inches, and speed $R$ is in feet per second.

(a) Find the equation of the regression line of speed as a linear function of length.

(b) Plot on this paper the data points and the regression line.

(c) What aspect of the plot in Part (b) shows that the data is not precisely linear?

(d) What aspect of the plot in Part (b) shows that the data can be approximated well by a linear function?

(e) Determine the slope of the regression line; be sure to indicate the units for the slope.

(f) Which animals run faster than predicted by the linear regression model? Which slower?

4. A bowl is taken from a freezer and is placed in a preheated oven. Its temperature is recorded as

| Minutes $t$ | 0 | 4 | 8 | 12 |
|---|---|---|---|---|
| Temperature $B$ | 0 | 61.90 | 102.52 | 129.16 |

where the temperature of the bowl $B$ is measured in degrees Fahrenheit.

The temperature difference is given by $D = 180 - B$.

(a) Fill in the missing parts of the following table

| Minutes $t$ | 0 | 4 | 8 | 12 |
|---|---|---|---|---|
| Temperature difference $D = 180 - B$ | 180 | | | |

(b) Find the linear regression line for $\ln D$ versus $t$.

(c) Based on the data from part (a) and the regression line for $\ln D$, explain why $D$ is an exponential function of $t$.

(d) Find a formula for $D$ as an exponential function of $t$.

(e) Find a formula for $B$ as a function of $D$ only.

(f) Combining formulas, find a formula for $B$ as a function of $t$ only.

(g) Graph $B$ as a function of $t$ for the first 40 minutes.

(h) What is the concavity of the graph of $P$ (concave up or concave down) ? Explain what this means in terms of the temperature of the bowl.

(i) What is the temperature of the freezer?

(j) What is the temperature of the oven?

5. A tank of water is being drained. Here is the depth of water $W$ in inches at various $t$, minutes since the draining began:

| Time $t$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Water depth $W$ | 32 | 24.5 | 18 | 12.5 | 8 |

   (a) Show that $W$ can be modeled as a quadratic function of $t$.

   (b) Write a formula for $W$ as a quadratic function of $t$.

   (c) How long will it take for the tank to be completely drained?

   (d) Is $W$ increasing or decreasing?

   (e) Is $\dfrac{dW}{dt}$ positive or negative?

   (f) Explain why $\dfrac{dW}{dt}$ is linear.

   (g) Carefully make three graphs:

      i. Graph $W$ as a function of $t$ for $t$ from zero until the tank is drained.

      ii. Sketch a graph of $\dfrac{dW}{dt}$ as a function of $t$ for the same interval.

      iii. Sketch a graph of the rate of change of $\dfrac{dW}{dt}$, that is, the second order rate of change of $W$, as a function of $t$ for the same interval.

6. The tables below show linear, quadratic, or exponential data. Determine which is which, explain your reasoning and write a formula for the function in each case.

Table A

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $f(t)$ | 6.7 | 7.77 | 9.02 | 10.46 | 12.13 | 14.07 |

Table B

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $g(t)$ | 0 | 7.7 | 17.24 | 28.62 | 41.84 | 56.90 |

Table C

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $h(t)$ | 5.8 | 7.53 | 9.26 | 10.99 | 12.72 | 14.45 |