Practice Problems for the Final Exam, Math 2163, Spring 2009

1. Consider the following triple integral written in spherical coordinates as follows

$$I = \int_{\pi/2}^{\pi} \int_{0}^{\pi/2} \int_{0}^{1} \rho^3 \sin\phi \cos\phi \; d\rho \, d\phi \, d\theta.$$

Express the integral I in Cartesian coordinates by replacing appropriately all the question marks below

$$I = \int_{?}^{?} \int_{?}^{?} \int_{?}^{?} ? \; dz \, dx \, dy.$$

Do not evaluate.

2. Use an appropriate change of variables to compute the double integral

$$\int\int_R (x+y)\, dA,$$

where R is the parallelogram in the xy-plane with vertices $(0,0), (3,1), (5,-1), (2,-2)$.

3. Find the volume of the solid (the "ice-cream cone") that lies within the sphere $x^2+y^2+z^2 = 4z$ and inside the cone $z = \sqrt{3(x^2+y^2)}$.

4. a) Compute the work W done by the force field $\mathbf{F} = -xy\mathbf{i} + x^2\mathbf{j}$ on a particle that moves counterclockwise along the arc C_1 of the circle $x^2+y^2 = 2$ starting at $(1,1)$ and ending at $(-1,-1)$.

b) Compute the work done by the same force field on a particle that moves along the line segment C_2 starting at $(1,1)$ and ending at $(-1,-1)$.

c) Is the vector field $\mathbf{F}$ conservative in $\mathbb{R}^2$? Explain your answer.

5. Evaluate the line integrals $\int_C x^2\, ds$ and $\int_C x^2\, dy$, where C is the line segment from $(1,0)$ to $(2,3)$.

6. Use Green's theorem to compute the line integral $\int_C x^2y\, dx - xy^2\, dy$, where C is the circle $x^2+y^2 = 4$ with the clockwise orientation.

7. Consider the vector field $\mathbf{F}(x,y) = (3x^2)\,\mathbf{i} + (\frac{z^2}{y})\,\mathbf{j} + (2z\ln y)\,\mathbf{k}$.

a) Determine the maximal domain of definition D of $\mathbf{F}$ find a potential function $f(x,y,z)$ for $\mathbf{F}(x,y)$ on D such that $f(0,1,1) = 1$.

b) Compute the line integral

$$\int_C \mathbf{F}\cdot d\mathbf{r}$$

where the curve C consists of the segments AB, BD in this order, where $A(2,1,3)$, $B(2,7,1)$, $D(1,2,2)$.

8. Consider the vector field $\mathbf{F}(x,y) = (2xy+1)\,\mathbf{i} + (x^2+2x)\,\mathbf{j}$.

a) Compute the line integral $\int_{C_1} \mathbf{F}\cdot d\mathbf{r}$ where C_1 is the line segment from $(-1,0)$ to $(1,0)$.

b) Use Green's theorem to compute the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where the closed curve C consists of the line segment C_1 from $(-1,0)$ to $(1,0)$ followed by the curve C_2, which is the counterclockwise top half of the circle $x^2+y^2 = 1$ from $(1,0)$ to $(-1,0)$.

c) Use parts a) and b) to find the value of $\int_{C_2} \mathbf{F}\cdot d\mathbf{r}$.

9. Evaluate $\int\int\int_E x^2\, dV$, where E is the solid that lies whitin the cylinder $x^2+y^2=2$ above the plane $z=1$, and below the cone $z^2=4x^2+4y^2$.

10. Compute the line integrals $\int_{C_1} \mathbf{F}\cdot d\mathbf{r}$ and $\int_{C_2} \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = \frac{-y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j}$ and the closed contours C_1 and C_2 are pictured below.