

Practice Problems for Midterm 1, Math 2163, Spring 2009

1. Let $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = \mathbf{j} + \mathbf{k}$ be two vectors in space. Compute the angle between \mathbf{a} and \mathbf{b} , and two unit vectors perpendicular to both \mathbf{a} and \mathbf{b} .

2. Find the distance between the two lines with equations $x = 2+t, y = 1-t, z = 6+3t$ and $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, with $\mathbf{r}_0 = \langle 1, 0, -2 \rangle$ and $\mathbf{v} = \langle 2, 1, -1 \rangle$.

3. Find the parametric equations of the line of intersection of the planes

$$2x + y - 3z = -4$$

and

$$x - y + 2z = 1.$$

4. Find an equation of the plane that contains the points $(0, 1, 2), (-1, 2, 3)$ and $(1, 1, 2)$.

5. Draw a contour map for the function $f(x, y) = \frac{y^2}{x+1}$.

6. Let $F(x, y, z)$ be a function of three variables such that

$$F_x(3, 5, 2) = 1, \quad F_y(3, 5, 2) = 2, \quad F_z(3, 5, 2) = 3.$$

Assume that

$$x(u, v) = u + v, \quad y(u, v) = u^2 + v^2, \quad z(u, v) = uv.$$

Let $f(u, v) = F(x(u, v), y(u, v), z(u, v))$. Calculate

$$f_u(1, 2) \quad \text{and} \quad f_v(1, 2).$$

7. Find all second order partial derivatives of the function $f(x, y) = y^2 \cos(3x - y)$.

8. The two sides of a rectangle are measured to be 3m and 5m, with a possible error of 3cm. Use differentials to estimate the maximum error in calculating the diagonal of the rectangle and its area.

9. Use implicit differentiation to find $\partial z / \partial x$ and $\partial z / \partial y$ given that $x + \sin(xz^2) = yz$.

10. Find an equation for the tangent plane to the surface $\{(x, y, z) : xyz = 6\}$ at $(1, 2, 3)$.