Practice Problems for Midterm 2, Math 2163, Spring 2009

1. Find the largest product the positive numbers $x, y, z > 0$ can have, if

$$x + y + z^2 = 20.$$

2. Evaluate the integral

$$I = \int_0^1 \int_{x^2}^1 xe^{-y^2}\, dy\, dx.$$

3. A flat triangular metal plate is represented by a region in the xy–plane bounded on the left by the line $x = -2$, on the bottom by the line $y = -2$, and on the right by the line $y = -x$. If the temperature at any point is given by $T(x, y) = x^2y - 4y - 4x + 50$, find the hottest and coolest points on the plate.

4. Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy–plane.

5. An ant is climbing a dirt hill whose shape is a paraboloid given by the equation $z = 100 - 2x^2 - y^2$, where x, y and z are measured in inches. The positive x-axis points east and the positive y-axis points north.

a) If the ant starts moving northwest from $(2, 3, 83)$, will it start to ascend or descend? Why?

b) In which direction should the ant start moving from $(2, 3, 83)$ so that it experiences neither an instantaneous ascend nor descend? Your answer should be expressed as a unit direction vector in $\mathbb{R}^2$.

6. Rewrite the iterated triple integral

$$\int_0^2 \int_0^{4-z^2} \int_0^{2-z} f(x, y, z) dx\, dy\, dz.$$

as the iterated triple integral

$$\int_?^? \int_?^? \int_?^? f(x, y, z) dz\, dx\, dy.$$

7. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane and inside the cylinder $x^2 + y^2 = 4x$.

8. Find the local maximum and minimum values and saddle points of the function $f(x, y) = x^4 + y^4 - 4xy + 2$.

9. Compute the following triple integral

$$\int\int\int_E y\, dV,$$

where E is the solid region enclosed by the coordinate planes and the planes $x = 2, y = 1$ and $x + 2y + z = 4$ in the first octant.

10. Find the area of the region within both of the circles $r = \cos\theta$ and $r = \sin\theta$.