Math 20B, Exam 1, 4/27/2011 **Name:** ____________________

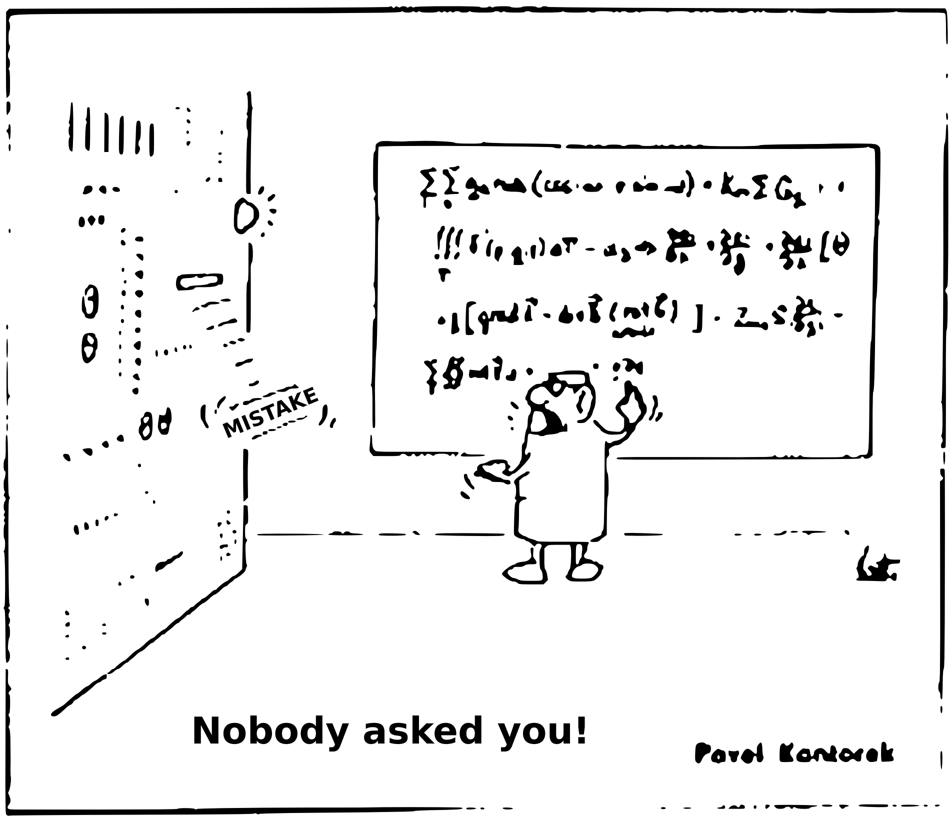
Section (the section you attend): ____________________

There are 7 pages and 6 questions, for a total of 100 points.

No calculators, no books.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Unless otherwise stated, show all your work for full credit.

Good luck!!



Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	20	0	100
Score:							

1. (20 points) No need to explain how you got the answer on this question.

   (a) Assume $\int_0^5 f(x)dx = 5$ and $\int_0^5 g(x)dx = -3$.

   Find: $\displaystyle\int_0^5 \big(f(x) + g(x)\big)dx$

   Solution: $5 + (-3) = 2$

   (b) Compute $\displaystyle\int_{-1}^{0} (x+1)^{500} dx$

   Solution: Use substitution $u = x + 1$, so $du = dx$.

$$\int_{-1}^{0} (x+1)^{500} dx = \int_0^1 u^{500} du = \left[\frac{u^{501}}{501}\right]_{u=0}^{1} = \frac{1}{501}$$

   (I told you I would ask this question!)

   (c) Compute $(\cos(\pi/4) + i\sin(\pi/4))^{40}$. Write answer as $a + ib$ without any sines and cosines.

   Solution:

$$\begin{aligned}(\cos(\pi/4) + i\sin(\pi/4))^{40} &= \cos(40\pi/4) + i\sin(40\pi/4) = \cos(10\pi) + i\sin(10\pi) \\ &= \cos(0) + i\sin(0) = 1 \qquad \text{(that is } 1 + 0i\text{)}\end{aligned}$$

   (d) Write the equation $r\cos(\theta) = r\sin(\theta)$ in rectangular (cartesian) coordinates x and y.

   Solution: $x = y$

   (e) Give the formula for the average value of a function $f(x)$ on the interval $[a, b]$ (as in the definition in the book)

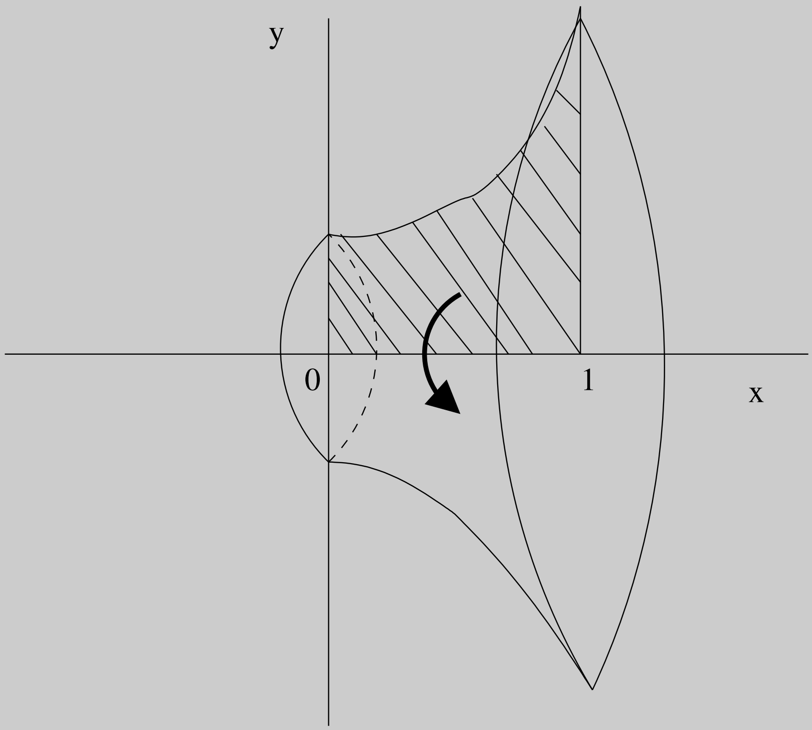
   Solution:

$$\text{Average value} = \frac{1}{b-a}\int_a^b f(x)dx$$

   (see page 387)

2. (20 points) Find the volume of a solid obtained by rotation of the region between $y = x^3 + 1$ and the x-axis from $x = 0$ to $x = 1$, rotated around the x-axis. Make sure to sketch the setup. *Points off for no sketch!*

Solution:

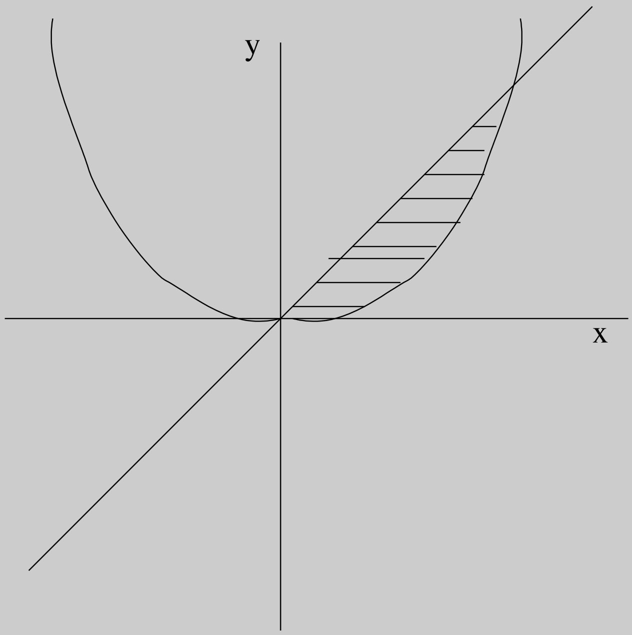


$$\text{Volume} = \int_0^1 \pi(x^3+1)^2 dx = \pi \int_0^1 (x^6 + 2x^3 + 1)dx$$

$$= \pi \left[\frac{x^7}{7} + \frac{1}{2}x^4 + x\right]_{x=0}^1 = \pi\left(\frac{1}{7} + \frac{1}{2} + 1\right) \qquad \text{(ok to leave like this)}$$

3. (20 points) Find area between $y = x^2$ and $y = x$. Make sure to sketch the setup. *Points off for no sketch!*

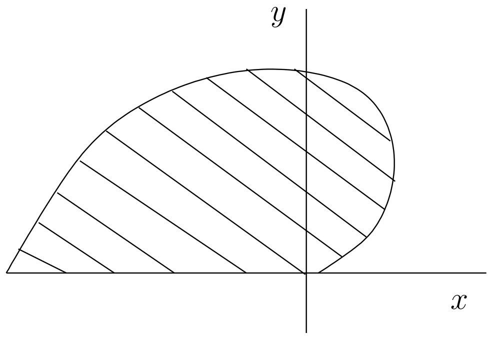
Solution:



First $x = x^2$ when $x = 0$ or $x = 1$, so

$$\int_0^1 (x - x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{x=0}^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

4. (20 points) Find area of the shaded region, where the curve is given in polar coordinates by $r = 2\theta$.



Solution:

$$\frac{1}{2}\int_0^{\pi} (2\theta)^2 d\theta = \frac{4}{2}\int_0^{\pi} \theta^2 d\theta = 2\left[\frac{\theta^3}{3}\right]_{\theta=0}^{\pi} = \frac{2\pi^3}{3}$$

5. (20 points) Solve using substitution:

$$\int x^2\sqrt{x^3+1}\,dx$$

Solution: Use: $u = x^3 + 1$, $du = 3x^2 dx$

$$\int x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\frac{2}{3}u^{3/2} + C = \frac{2}{9}(x^3+1)^{3/2} + C$$

6. (5 points (bonus)) Compute $\cos(3+i)$. Write as $a+ib$. *(No partial credit here, it's a bonus)*

Solution: Use $\cos(z) = \frac{e^{iz}+e^{-iz}}{2}$, you will obtain:

$$\frac{\cos(3)(e+e^{-1})}{2} - i\frac{\sin(3)(e-e^{-1})}{2}$$