Math 20B, Exam 2, 5/25/2011 **Name:** _____

Section (the section you attend)**:** _____

There are 7 pages and 6 questions, for a total of 100 points.

No calculators, no books.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Unless otherwise stated, show all your work for full credit.

Good luck!!

"This, my boy, is a brain.
It does terrible things
to people."

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	0	
Total:	100	

1. (20 points) No need to explain how you got the answer on this question.

   (a) Does $\int_1^\infty \frac{20}{x} dx$ converge?

   Solution: No as $\int_1^\infty \frac{1}{x} dx$ does not converge

   (b) Does $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$ converge?

   Solution: Yes, $\pi > 1$.

   (c) What are the poles of $\frac{x+1}{(x-1)(x+3)}$

   Solution: 1 and -3

   (d) Find $\lim_{n\to\infty} \frac{4n^2 - n}{n^2 + n + 1}$

   Solution: 4

   (e) $\int e^{ix} dx$

   Solution: $\frac{e^{ix}}{i} = -ie^{ix}$

2. (20 points) Evaluate $\displaystyle\int_{-\infty}^{1} e^{3t}\,dt$ if it converges or (if it does not) justify why not.

Solution: $\displaystyle\int_{-\infty}^{1} e^{3t}\,dx = \lim_{R\to-\infty}\int_{R}^{1} e^{3t}\,dx = \lim_{R\to-\infty}\left[\frac{e^{3t}}{3}\right]_{x=R}^{1} = \lim_{R\to-\infty}\left[\frac{e^3}{3} - \frac{e^{3R}}{3}\right]_{x=R}^{1} = \frac{e^3}{3}$

3. (20 points) Take $\int_{-2}^{2} x^2 \, dx$. Compute T_4 (the trapezoid rule with 4 intervals).

Solution: $T_4 = \frac{1}{2}\left((-2)^2 + 2(-1)^2 + 2(0)^2 + 2(1)^2 + 2^2\right) = 6$

4. (20 points) Solve $\displaystyle\int \frac{1}{(x+1)(x-1)}\,dx$

Solution:

First: $\dfrac{1}{(x+1)(x-1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1} = \dfrac{A(x-1)+B(x+1)}{(x+1)(x-1)} = \dfrac{(A+B)x+(B-A)}{(x+1)(x-1)}$

So $A+B=0$ and $B-A=1$. That is $A=-B$ and so $2B=1$ and so $B=1/2$ and $A=-1/2$ and

So $\displaystyle\int \frac{1}{(x+1)(x-1)}\,dx = \frac{-1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx = \frac{-1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$

5. (20 points) Solve $\int 3x \sin(4x)\ dx$

Solution:

$$\int 3x \sin(4x)\ dx = 3x\frac{-\cos(4x)}{4} - \int 3\frac{-\cos(4x)}{4}\ dx = \frac{-3}{4}x\cos(4x) + \frac{3}{16}\sin(4x)$$

6. (5 points (bonus)) Compute $\sum_{n=1}^{k} \ln \frac{n}{n+1}$ *(No partial credit here, it's a bonus) Hint: remember rules of logarithms*

Solution: It's a telescoping sum!

$$\begin{aligned}\sum_{n=1}^{k} \ln \frac{n}{n+1} &= \sum_{n=1}^{k} (\ln n - \ln(n+1)) \\ &= (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + \cdots + (\ln k - \ln(k+1)) \\ &= -\ln(k+1)\end{aligned}$$