All the homework problems and even the unassigned problems are good sample problems. This is just a list of possible extra problems

1. Find steady state solution to $u_t = u_{xx}$, $u_x(0,t) = u_x(1,t) = 0$, $u(x,0) = x^2$.

2. Use separation of variables to solve

$$\begin{array}{rll} \text{(PDE)} & u_t = u_{xx} & 0 < x < \pi, \quad t > 0, \\ \text{(BC)} & u(0,t) = 0, \quad u(\pi,t) = 0, & t > 0, \\ \text{(IC)} & u(x,0) = \sin(3x) - 3\sin(5x) & 0 < x < \pi. \end{array}$$

3. Convert the problem to one with homogeneous BC

$$\begin{array}{rll} \text{(PDE)} & u_t = u_{xx} & 0 < x < \pi, \quad t > 0, \\ \text{(BC)} & u_x(0,t) = 1, \quad u_x(\pi,t) = 0, & t > 0, \\ \text{(IC)} & u(x,0) = \cos(x) & 0 < x < \pi. \end{array}$$

4. Find Fourier series for $f(x) = x^2$ on $[-1,1]$ (extended to a 2-periodic function).

5. Solve

$$\begin{array}{rll} \text{(PDE)} & u_t = -5u_x & 0 < x < \infty, \quad t > 0, \\ \text{(BC)} & u(0,t) = 1, & t > 0, \\ \text{(IC)} & u(x,0) = e^{-x}, & 0 < x < \infty. \end{array}$$

   using Laplace transform.

6. Find Fourier transform of

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

7. Solve

$$\begin{array}{rll} \text{(PDE)} & u_t = u_{xx} + t\sin(2\pi x) & 0 < x < 1, \quad t > 0, \\ \text{(BC)} & u(0,t) = 0, \quad u(1,t) = 0, & t > 0, \\ \text{(IC)} & u(x,0) = \sin(\pi x), & 0 < x < 1. \end{array}$$

8. Find the 2-periodic Fourier series of the 2-peridic functions:
   a) $f(x) = 3\sin(2\pi x) - \cos(4\pi x)$
   b) $f(x) = \sin^2(\pi x)$

9. Solve

$$\begin{array}{rll} \text{(PDE)} & \nabla^2 u = 0 & \text{in the unit circle,} \\ \text{(BC)} & u(1,\theta) = \sin^2(\theta) & \end{array}$$

10. Solve

$$\begin{array}{rll} \text{(PDE)} & u_{tt} = u_{xx} & -\infty < x < \infty, \quad t > 0, \\ \text{(IC)} & u(x,0) = \sin(x), \quad u_t(x,0) = \cos(x) & \end{array}$$

11. Use separation of variables to solve

$$\begin{array}{rll} \text{(PDE)} & u_{tt} = u_{xx} & 0 < x < \pi, \quad t > 0, \\ \text{(BC)} & u(0,t) = 0, \quad u(\pi,t) = 0, \quad t > 0, & \\ \text{(IC)} & u(x,0) = \sin(3x) - 3\sin(5x), \quad u_t(x,0) = 0 \quad 0 < x < \pi. & \end{array}$$

12. Convert the problem to a dimensionless one

$$\begin{array}{rll} \text{(PDE)} & u_t = 4u_{xx} & 0 < x < \pi, \quad t > 0, \\ \text{(BC)} & u(0,t) = 3, \quad u(\pi,t) = -1, \quad t > 0, & \\ \text{(IC)} & u(x,0) = \cos(x) \quad 0 < x < \pi. & \end{array}$$

13. Find finite Fourier sine series for $f(x) = x^2$ on $[0,1]$.

14. Classify $u_{xx} - u_{xy} = u_x$, and put it into cannonical form.

15. Classify
 a) $u_{xy} - u_{xx} + u_{yy} + u_x - u = 0$.
 b) $u_{xx} + u_{yy} + u_x - u = 0$.
 c) $u_{xx} + u_{yy} - 8u_{xy} + u_y - u = 0$.
 d) $u_{xx} + u_y - u = 0$.

16. Suppose $u(x,t)$ for $-\infty < x < \infty$, $t > 0$, satisfies $u_{tt} = 0$, $u(x,0) = \sin(x)$, $u_t(x,0) = \cos(x)$. Find u.

17. Suppose $v_{xx}+v_{yy} = xy$, and $w_{xx}+w_{yy} = 1$. Using v and w, find a solution $u(x,y)$ of $u_{xx}+u_{yy} = 3xy-5$.

18. Solve

$$\begin{array}{rll} \text{(PDE)} & \nabla^2 u = 0 & \text{in the square } 0 < x < 1,\ 0 < y < 1, \\ \text{(BC)} & u = xy & \text{on the boundary} \end{array}$$

19. Take

$$\begin{array}{rll} \text{(PDE)} & u_t = 3u_{xx} & \text{where } 0 < x < 2,\ t > 0, \\ \text{(BC)} & u(0,t) = 0, u_x(2,t) = 0 & \\ \text{(IC)} & u(x,0) = 1 & \end{array}$$

 Using the explicit finite difference method with $\delta x = h = 0.5$, and $\delta t = k = 0.1$, find the approximate values of the solution at $t = 0.2$.

20. Using the Galerkin method using FEM with just two nodes 0 and 1, find the approximate solution to $u' + u = 1$, $u(0) = 3$.

21. Using the method of characteristics, solve

$$\begin{array}{rll} \text{(PDE)} & 2u_x + u_t + 2u = 0 & \infty < x < \infty, \quad t > 0, \\ \text{(IC)} & u(x,0) = \sin(x) & -\infty < x < \infty. \end{array}$$

22. Solve

$$\begin{array}{rll} \text{(PDE)} & u_t + 2uu_x = 0 & \infty < x < \infty, \quad t > 0, \\ \text{(IC)} & u(x,0) = 3x & -\infty < x < \infty. \end{array}$$

23. Guess solutions to the following problems (possibly challenging):
a)

$$\begin{aligned} &\text{(PDE)} \qquad u_t + \sin(u)u_x = 0 \qquad \infty < x < \infty, \quad t > 0, \\ &\text{(IC)} \qquad u(x,0) = 2 \quad -\infty < x < \infty. \end{aligned}$$

b)

$$\begin{aligned} &\text{(PDE)} \qquad u_t + uu_x = 1 \qquad \infty < x < \infty, \quad t > 0, \\ &\text{(IC)} \qquad u(x,0) = 0 \quad -\infty < x < \infty. \end{aligned}$$

24. What does Huygen's principle say?

25. (challenging) Suppose that $u_y + u_x = 0$ on the unit square. Can you find boundary conditions so that the problem is not well-posed? That is, find boundary conditions so that the problem is not solvable. Make your boundary condition a continuous function.

26. Solve intuitively: Suppose I have a very long pipe of inner radius 1 meter and outer radius 1.5 meters. The pipe is in siberia so outside temperature is exactly 0 degrees. We are running hot water through the pipe at temperature 100 degrees. What is the steady state temperature of the pipe material. Use polar coordinates to describe the temperature.

27. Change $u_{xy} - u_y + u = 0$ into polar coordinates.

28. Let u be a harmonic function, that is $\nabla^2 u = 0$. Suppose on circle of radius 0.1, the value of the function is $1 + \sin\theta$. What is the value of u at the origin.

29. Using finite difference method with $h = k = 0.5$, what is the approximate value of $u(0.5, 0.5)$ if $\nabla^2 u = u$ on the unit square $0 < x, y < 1$, where boundary conditions are $u(0, y) = y$, $u(1, y) = 1-y$, $u(x, 0) = x$, $u(x, 1) = 1 - x$.