

### Review problems for Exam 1.

1. Suppose  $x$  is an element of a group  $G$ . and  $|x| = n$ . If  $n = ts$  for some positive integers  $s$  and  $t$ , prove that  $|x^t| = s$ .
2. Show that  $\langle a, b | a^2 = b^2 = (ab)^n = 1 \rangle$  gives another presentation for  $D_{2n} = \langle r, s | r^n = s^2 = 1, rs = sr^{-1} \rangle$  in terms of the generators  $a = s, b = sr$ . (Construct a well-defined isomorphism.)
3. Find all numbers  $n$  such that  $S_n$  contains an element of order  $n$ .
4. Show that if  $n$  is not prime then  $\mathbf{Z}/n\mathbf{Z}$  is not a field.
5. Prove that  $D_{24}$  and  $S_4$  are not isomorphic.
6. Assume  $n$  is an even positive integer and show that  $D_{2n}$  acts on the set of consisting of pairs of opposite vertices of a regular  $n$ -gon. Find the kernel of this action.
7. Prove that  $G$  cannot have a subgroup  $H$  with  $|H| = n - 1$ , where  $n = |G|$ .
8. Let  $H_1 \leq H_2 \leq \dots$  be an ascending chain of subgroups of  $G$ . Prove that  $\cup_{i=1}^{\infty} H_i$  is a subgroup of  $G$ .
9. Prove that  $\mathbf{Z} \times \mathbf{Z}$  is not cyclic.
10. Find a presentation for  $\mathbf{Z}/n\mathbf{Z}$  with one generator.
11. Prove that the subgroup of  $S_4$  generated by (12) and (13)(24) is isomorphic to the dihedral group of order 8.
12. Prove that the subgroup generated by any two distinct elements of order 2 in  $S_3$  is all of  $S_3$ .
13. Prove that the rational numbers  $\mathbf{Q}$  is not a finitely generated group.