Section 2.3
Complex Numbers

Our Number System:

_____ Numbers:

Integers: $0, \pm 1, \pm 2, \pm 3, \cdots$

Rational numbers: fractions

Irrational numbers: $\pi, \sqrt{2}, \ e$ etc. (non-ending, non-repeating decimals)

_____ Numbers: numbers that involve _____ (see below)

_____ Numbers $= \begin{cases} Real\ Numbers \\ + \\ Imaginary\ Numbers \end{cases}$

Example 1: Solve:

a. $x^2 - 4 = 0$

b. $x^2 + 4 = 0$

Imaginary numbers involve $\sqrt{-1}$

We define _____

Notice that:

The Standard Form of a Complex Number:

Examples: $-2 + 7i$, $3 - 0.5i$, $\frac{2}{3} - \frac{3}{5}i$

If $b = 0$ it's just a real number : $4 + 0i = 4$

If $a = 0$ it's called a _____ imaginary number: $0 + 5i = 5i$

Add or Subtract Complex Numbers:
To add or subtract complex numbers, you add or subtract the real and imaginary parts separately.

Always write the answer in Standard Form $(a \pm bi)$

Example 2: Add or subtract complex numbers.

a. $(-2+6i)+(4+3i)=$

b. $(-2+6i)-(3-4i)=$

Multiply Complex Numbers:
To multiply complex numbers, you use the FOIL technique. You will probably also need to use the fact that $i^2 = -1$

Always write the final answer in Standard Form

Example 3: Multiply:

a. $(-2+6i)(3-4i)=$

b. $(2-3i)^2=$

c. $(3-4i)(3+4i)=$

In the previous example: $3-4i \ \& \ 3+4i$ are call _____ of each another.

Two complex numbers are called conjugates of each other if the real and imaginary parts are the same but the sign between them is different.

Example 4: Write the complex conjugate of the complex number.

a. $2-5i$ b. $-5+7i$ c. $-9i$ d. -14

Multiplying a complex number by its conjugate produces a real number.
In a previous example: $(3-4i)\ (3+4i)=25$

In general,

$$\begin{aligned}&(a-bi)(a+bi)\\&=a^2+abi-abi-b^2i^2\\&=a^2-b^2i^2\\&=a^2-b^2(-1)\\&=a^2+b^2\end{aligned}$$ (a real number)

Simplifying a quotient (fraction) involving complex numbers

Example 5: Simplify and write answer in standard form: $\frac{2+3i}{3-4i}$

Technique: Multiply the numerator and denominator by the _____ of the denominator (a "well-chosen" one) :

$$\frac{2+3i}{3-4i}\cdot\frac{3+4i}{3+4i}$$

Example 6: Perform the indicated operation and write answer in standard form:

$$\frac{2}{1+i}-\frac{3}{1-i}$$