

Section 5.2
Systems of Linear Equations in Two Variables

Recall that a system of equations is two or more equations considered together. (See Section 5.1.)
In particular, if all of those functions are linear, we call it a _____ of _____.

Ex.
$$\begin{cases} 3x - 2y = 5 \\ x + 2y = 7 \end{cases}$$

We studied the Substitution Method in Section 5.1. Now we will learn the _____ Method.

Solving by Elimination:

1. Arrange the equations so that they have the same form – the variables are lined up
2. Multiply one or both equations by constants so that x or y has the same coefficient (or different signs)
3. Add the equations. One variable is **eliminated**. Solve for the remaining variable.
4. Back-substitute in one of the equations and solve for the other variable

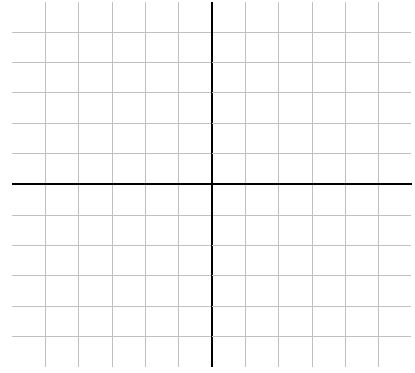
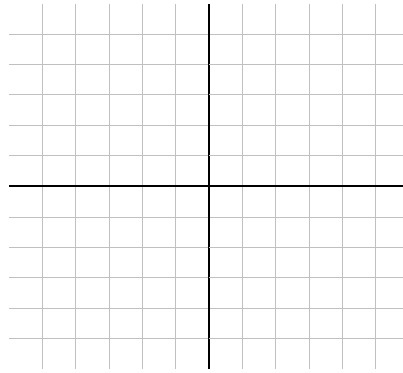
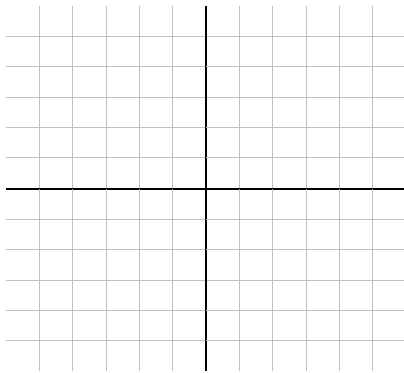
Example 1: Solve the following systems of equations.

a.
$$\begin{cases} 3x - 2y = 5 \\ x + 2y = 7 \end{cases}$$

b.
$$\begin{cases} 3x + 2y = 10 \\ 2x = -5y + 3 \end{cases}$$

Recall that a _____ of a system of equations is an _____ that is a solution of _____ equations.

Ex.



Example 2: Solve $\begin{cases} 3x - 4y = 8 \\ 6x - 8y = 16 \end{cases}$

Let us see the above example again. We notice that the equation (2) divided by 2 gives us the equation (1). This is the case of a dependent system of equations. If you notice a system of equations is dependent, there is a simpler method for solving it. Let us try Example 2 again.

Example 2': Solve $\begin{cases} 3x - 4y = 8 \\ 6x - 8y = 16 \end{cases}$

Example 3: Determine which system of equations are dependent.

a.
$$\begin{cases} 10x - 5y = 8 \\ 30x - 15y = 24 \end{cases}$$

b.
$$\begin{cases} 2x - 5y = 7 \\ 4x + 10y = 14 \end{cases}$$

c.
$$\begin{cases} 5x + 2y = 2 \\ y = -\frac{5}{2}x + 1 \end{cases}$$

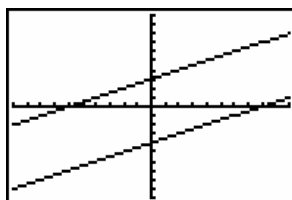
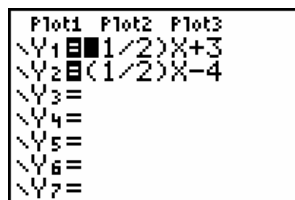
d.
$$\begin{cases} 2x + 6y = 4 \\ 5x + 15y = 10 \end{cases}$$

e.
$$\begin{cases} y - \frac{1}{2}x - 3 = 0 \\ x - 2y = 8 \end{cases}$$

Let us see Example 3, e. one more time. Notice that the left-hand side of the second equation is obtained as three times the left-hand side of the first equations. But this is not a dependent system of equations, because the constant term cannot be obtained in the same way. This is the case of inconsistent.

Example 4: Solve
$$\begin{cases} y - \frac{1}{2}x - 3 = 0 \\ x - 2y = 8 \end{cases}$$

Graphically:



Example 5: Solve:

a.
$$\begin{cases} 4x - 2y = 9 \\ 2x - y = 3 \end{cases}$$

b.
$$\begin{cases} \frac{3}{5}x - \frac{2}{3}y = 7 \\ \frac{2}{5}x - \frac{5}{6}y = 7 \end{cases}$$