Matrices and Systems of Equations

Matrices:

If m and n are positive integers, an ______ __________ is a rectangular array

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

in which each a_{ij}, called ___________, of the matrix is a real number.

An $m \times n$ matrix has ___ rows and ___ columns.

NOTE: a_{ij} is the entry in the ____ row and _____ column. (Ex. a_{53} is in the ______________________)

- A matrix having m rows and n columns is said to be of __________ of _________.
- If __________, the matrix is called a __________ matrix of order ___.
- For a square matrix, the entries ____ (ex. ____, ____, ____) are the ________ ___________ entries.

Example 1: Determine the order of each matrix.

a. $\begin{bmatrix} 5 & 0 & -2 & -1 & 4 \\ 2 & 13 & -5 & 7 & 2 \\ -3 & 1 & 3 & 0 & -11 \end{bmatrix}$ b. $\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & -6 \end{bmatrix}$

The goal in this section is to solve the systems of linear equations by using matrices.

A matrix can be derived from a system of linear equations.

System

$$\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ 3x \qquad - z = 0 \end{cases}$$

Augmented Matrix

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & 6 \\ 2 & -1 & 1 & \vdots & 3 \\ 3 & 0 & -1 & \vdots & 0 \end{bmatrix}$$

Coefficient Matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 3 & 0 & -1 \end{bmatrix}$$

We can apply the elementary row operations to matrices.

Elementary Row Operations:

1. Interchange two rows.
2. Multiply a row by a nonzero constant.
3. Add a multiple of a row to another row.

Example 2: Compare the linear systems and matrix operations.

System	Associated Augmented Matrix
$\begin{cases} x+y+z=6 \\ 2x-y+z=3 \\ 3x \quad\quad -z=0 \end{cases}$	$\begin{bmatrix} 1 & 1 & 1 & \vdots & 6 \\ 2 & -1 & 1 & \vdots & 3 \\ 3 & 0 & -1 & \vdots & 0 \end{bmatrix}$

Gauss-Jordan Elimination:

Example 3: What is the corresponding system of a given matrix below?

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -4 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

Gauss-Jordan Elimination is a method to solve systems by continuing the reduction process until you obtained a matrix of the form, $\begin{bmatrix} 1 & 0 & 0 & \vdots & a \\ 0 & 1 & 0 & \vdots & b \\ 0 & 0 & 1 & \vdots & c \end{bmatrix}$ (where a, b, c are real numbers).

Example 4: Solve the system of linear equations by using Gauss-Jordan elimination.

a. $\begin{cases} x \qquad - 3z = -2 \\ 3x + y - 2z = 5 \\ 2x + 2y + z = 4 \end{cases}$

b. $\begin{cases} \qquad 2y - z = 14 \\ 7x - 5y \qquad = 6 \\ 2x - y + 3z = 24 \end{cases}$

Example 5: Solve the system of linear equations.

a. $\begin{cases} x \quad\quad + 4z = 1 \\ x + y + 10z = 10 \\ 2x - y + 2z = -5 \end{cases}$

b. $\begin{cases} 3x - 3y + 6z = 6 \\ x + 2y - z = 5 \\ 5x - 8y + 13z = 7 \end{cases}$