Section 6.1 - Part 1
Sequences

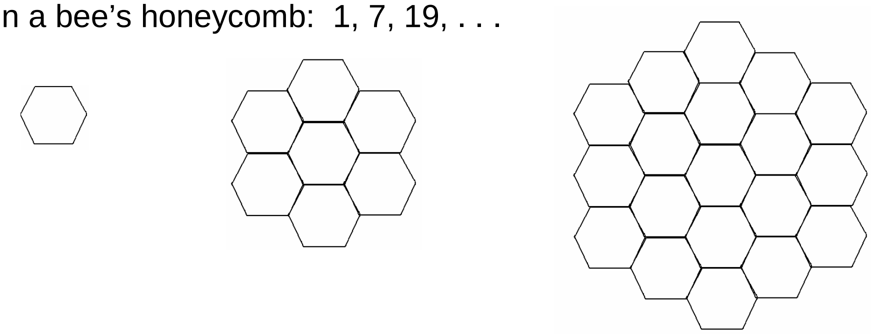
____________ is an ordered list of numbers.

Ex. 8, 15, 22, 29, 36, . . .

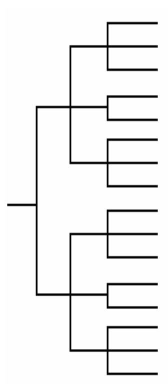
Sequences are very important in mathematics, statistics and sciences. Many things in nature or everyday life occur in sequences.

Ex.

1. Days in the months: 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31
2. Cells in a bee's honeycomb: 1, 7, 19, . . .



3. Number of descendants in a family tree



- each element of the sequence is called a ______.
- usually there is some pattern or formula that determines each term
- the terms are often referred to using subscripts: $a_1, a_2, a_3, \cdots$ (The *n*-th term is referred as a_n, and is read "*a* sub *n*")
- sequences have an index or counter variable (like *n*, *i* or *k*) that is used to refer to each term. Sequences usually start with an index of 1, but some start with 0.
- sequences can be ________ meaning they stop at some point, or _________ meaning the terms go on forever. Three dots, …, at the end, read "and so forth", shows that the sequence goes on infinitely.

Example 1: Write the first 5 terms of the sequence defined by: (Assume *n* begins with 1)

a. $a_n = 4n - 3$

b. $a_n = (-1)^n \dfrac{n}{n+1}$

Example 2: Find the indicated terms of the sequence $a_n = (-1)^n \dfrac{n+2}{n^2}$.

a. a_{10}

b. a_{25}

Example 3: Find the "apparent" *n*th term of each sequence.

a. $\dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dfrac{5}{6}, \dfrac{6}{7}, \dfrac{7}{8}, \cdots$

b. 8, 15, 22, 29, 36, . . .

<u>Factorials</u>:

If *n* is a positive integer, ___ ____________ is defined as

As a special case, zero factorial is defined as _________.

Sequences are sometimes defined using <u>factorials:</u> *n*! read "*n* factorial"

Example 4: Write the first 4 terms of the sequence defined by $a_n = (-1)^n \dfrac{n+1}{n!}$.

Example 5: Evaluate each factorial expression.

a. $\frac{9!}{7!}$

b. $\frac{14!}{3! \cdot 11!}$

c. $\frac{n!}{(n-3)!}$

<u>Series</u>:

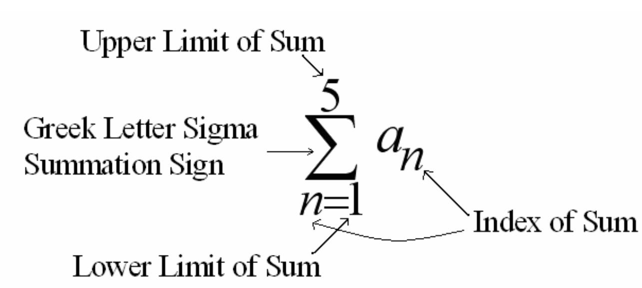
The ________ is the sum of the terms of a sequence:

Sequence:

Series:

<u>Sigma Notation</u>:

$$a_1 + a_2 + a_3 + a_4 + a_5 =$$



The index of the sum does not have to be n. It could be i or j or k or any letter.

These sums are really the same: $\sum_{n=1}^{10} a_n = \sum_{i=1}^{10} a_i = \sum_{k=1}^{10} a_k$

The <u>lower limit</u> of the sum doesn't have to be 1, the sum could start with any number.

Example 6: Write out the terms:

a. $\sum_{n=2}^{5} a_n =$

b. $\sum_{i=10}^{13} a_i =$

Example 7: Write out the terms and find the sum:

a. $\sum_{n=1}^{5} n =$

b. $\sum_{i=1}^{4} (3i-1) =$

c. $\sum_{k=-1}^{3} k^2 =$

Properties of Sums: (Let *c* be a constant.)

1. $\sum_{i=1}^{n} c =$

2. $\sum_{i=1}^{n} ca_i =$

3. $\sum_{i=1}^{n} (a_i + b_i) =$

Example 8: Find the sum.

a. $\sum_{i=1}^{7} 5 =$

b. $\sum_{k=1}^{100} 2 =$

c. $\sum_{i=1}^{5} 3i =$

d. $\sum_{n=2}^{4} 2n^2 =$

e. $\sum_{k=1}^{3} (k + k^2) =$

f. $\sum_{n=1}^{4} (3n-1) =$

Special Formulas:

1. $\sum_{i=1}^{n} i =$

2. $\sum_{i=1}^{n} i^2 =$

Example 9: Find the sum:

a. $\sum_{i=1}^{100} i =$

b. $\sum_{i=1}^{20} 4i =$

c. $\sum_{i=1}^{40} 3i^2 =$