EXAM 3
MATH 3013 SECTION 002, SPRING 2009

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Print Name and Student #

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(1) (10pts) Let u and v be vectors in an inner-product space, and suppose that $\|u\| = 3, \|v\| = 2$. Find $\langle u + 2v, u - 2v \rangle$.

(2) (10pts) Find the area of the triangle with vertices $(2, 1, -3), (3, 0, 4)$ and $(1, 0, 5)$ in R^3.

(3) (10pts) Let A be a 5×5 matrix with $\det A = 2$. Find the following.

(i) $\det(A + 2A)$

(ii) $\det(A^T \cdot A)$

(iii) $\det(A^{-2})$.

(4) (15pts) Let $A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & -3 & -2 \\ 2 & -2 & 3 \end{pmatrix}$.

(i) Find the cofactor of -3.

(ii) Compute $\det A$ by using pivots and row-operations **ONLY**.

(5) (10pts) (i) Find the adjoint of the matrix $\begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$.

(ii) Find the x_2 **only by using Cramer's rule** for

$$x_1 + 2x_2 = 1, \qquad 2x_1 + 3x_2 = -2.$$

(6) (10pts) Find the characteristic polynomial of $\begin{pmatrix} 1 & 0 & 0 \\ -8 & 4 & -6 \\ 8 & 1 & 9 \end{pmatrix}$, and eigenvalues of the matrix.

(7) (15pts) Let $A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 3 & 1 \end{pmatrix}$. The characteristic polynomial of A is given by $(\lambda+1)(\lambda-2)^2$.

(i) Find the algebraic multiplicities of $\lambda_1 = -1$ and $\lambda_2 = 2$.

(ii) Find the geometric multiplicities of $\lambda_1 = 1$ and $\lambda_2 = -3$.

(i) Is the matrix A diagonalizable ?

(8) (10pts) A matrix $A_{3\times 3}$ has an eigenvalue $\lambda_1 = 2$ with eigenvector $v_1 = (1, 1, 0)$, eigenvalue $\lambda_2 = -1$ with eigenvector $v_2 = (0, 1, 1)$ and eigenvalue $\lambda_3 = 3$ with eigenvector $v_3 = (1, 1, 1)$. Compute A^k.

(9) (10pts) Solve the system of linear differential equations:

$$\begin{aligned} x_1' &= x_1 - 3x_2 + 3x_3 \\ x_2' &= -5x_2 + 6x_3 \\ x_3' &= -3x_2 + 4x_3, \end{aligned}$$

where the matrix $\begin{pmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{pmatrix}$ has eigenvalues $1, 1, -2$.