EXAM 1
MATH 3613 SECTION 001, SPRING 2009

INSTRUCTOR: WEIPING LI

Print Name and Student #

SHOW WORK FOR CREDIT !!! MUST PROVIDE LOGICAL REASONS!!!

(1) (10pts) State the well-ordering axiom.

(2) (15pts) Prove that a and c leave the same remainder when divided by a positive integer n if and only if $a - c = nk$ for some integer k.

(3) (10pts) If $a|bc$ and $(a, b) = 1$, then prove that $a|c$.

(4) (20pts) Prove that $\sqrt{a}$ is rational if and only if a is a perfect square ($a = n^2$ for some integer n)

(5) (15pts) If $p \geq 5$ is prime, prove that $p^2 + 2$ is composite.

(6) (15pts) If $a, b, c, d \in Z$ are integers and $a = bc - d$, prove that $(a, b) = (b, d)$.

(7) (15pts) (a) If a, b, u, v are integers and $au + bv = 1$, prove that $(a, b) = 1$.

(b) Show by example that if $au + bv = d > 1$, then (a, b) may not be d.