1. If $n\%$ of 15 is 3.7 then what is 15% of n?

   (a) 55.5
   (b) 0.247
   (c) 3.7
   (d) 24.7
   (e) 4.05

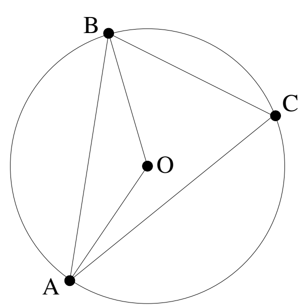
   Answer: C

2. Which of the following numbers is the largest?

   (a) $\sqrt{2} + \sqrt{15}$
   (b) $\sqrt{31}$
   (c) $\sqrt{5} + \sqrt{12}$
   (d) $\sqrt{6} + \sqrt{10}$
   (e) $\sqrt{3} + \sqrt{14}$

   Answer: C

3. In the diagram, triangle ABC is inscribed in circle O. If $\angle ABO = 20^\circ$ and $\angle CAO = 30^\circ$, find $\angle ACB$.



   (a) 50°
   (b) 70°
   (c) 90°
   (d) 130°
   (e) 140°

   Answer: B

4. A bicyclist goes up a hill at 10km/hr and back down the same hill at 40km/hr. What is the cyclist's average speed for the entire trip?

   (a) 20 km/hr
   (b) 25 km/hr
   (c) 30 km/hr
   (d) 12 km/hr
   (e) 16 km/hr

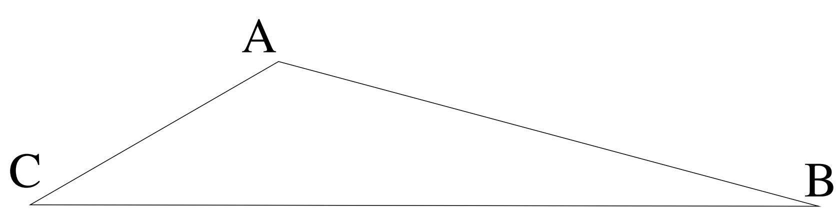
   Answer: E

5. For how many positive integers n is $n^2 + 1$ divisible by $n + 1$? (Zero is not positive.)

   (a) Infinitely many
   (b) None
   (c) More than two, but finitely many
   (d) One
   (e) Two

   Answer: D

6. In triangle ABC, $\angle A = 135°$, $AB = 1$, and $BC = \sqrt{2}$. Find $\angle C$.



   (a) 15°
   (b) 20°
   (c) 25°
   (d) 30°
   (e) 35°

   Answer: D

7. Say that a positive integer is *orderly* if its digits decrease when read left-to-right. (So 31 and 32 are orderly, but 33 and 34 are not.) How many positive integers are orderly? (One-digit numbers are all orderly. Zero is less than any other digit, so 10 is orderly, but 201 is not.)

   (a) 45
   (b) 1022
   (c) 512
   (d) 99
   (e) 3628800

   Answer: B

8. An arithmetic sequence $\{a_n\}$ has $a_1 = 8$ and $a_{1012} = 1100$. Find the sum

$$a_1 + a_2 + \cdots + a_{2022}$$

.

   (a) 2223108
   (b) 2224119
   (c) 2225130
   (d) 2226141
   (e) 2227152

   Answer: A

9. What is the probability that a randomly chosen divisor of a billion is a multiple of a million?

   (a) $\frac{9}{100}$
   (b) $\frac{4}{25}$
   (c) $\frac{16}{81}$
   (d) $\frac{2}{5}$
   (e) $\frac{1}{2}$

   Answer: B

10. For how many positive integers $n < 2022$ does the polynomial $x^2 + x - n$ factor as a product of two linear polynomials with integer coefficients?

    (a) None
    (b) 11
    (c) 22
    (d) 33
    (e) 44

    Answer: E

11. Nine dots are marked on the lattice points of a grid with x and y coordinates both in the set $\{1, 2, 3\}$. A triangle is constructed by choosing three (non-collinear) dots for its vertices,and the triangle's perimeter is computed. How many possible values could the perimeter have?

    (a) 8
    (b) 9
    (c) 10
    (d) 11
    (e) 12

    Answer: A

12. Consider the 20×22 grid of fractions below:

$$\begin{array}{ccccc} 1/1 & 1/2 & 1/3 & \cdots & 1/22 \\ 2/1 & 2/2 & 2/3 & \cdots & 2/22 \\ 3/1 & 3/2 & 3/3 & \cdots & 3/22 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 20/1 & 20/2 & 20/3 & \cdots & 20/22 \end{array}$$

    There are $20 \times 22 = 440$ entries in this grid, but some numbers appear multiple times. (For example, $1/1$ and $2/2$ are equal.) How many distinct numbers appear in the grid?

    (a) 261
    (b) 266
    (c) 271
    (d) 276
    (e) 281

    Answer: D

13. 2022 distinct lines are drawn across the plane, dividing it into R regions. Which of the following could be equal to R?

    (a) 3,648
    (b) 364,800
    (c) 36,480,000
    (d) 3,648,000,000
    (e) 36,480,000,000

    Answer: B

14. A cardboard box in the shape of a rectangular prism has exterior dimensions 4cm $\times$ 5cm $\times$ 6cm. If the walls (including the floor and ceiling) of the box have uniform thickness 1cm, find the volume of the cardboard. (Find the volume of the walls, not the interior of the box.)

    (a) 72
    (b) 96
    (c) 108
    (d) 114
    (e) 148

    Answer: B

15. A positive integer $n > 1$ is called *prime-seeming* if n is composite, but also not divisible by 2, 3, or 5 (any of the small primes one might most easily check in one's head). The three smallest prime-seeming numbers are $49 = 7^2$, $77 = 7 \cdot 11$, and $91 = 7 \cdot 13$. There are 306 prime numbers less than 2022. How many prime-seeming numbers are there less than 2022? (The number 1 is neither prime nor composite.)

    (a) 168
    (b) 202
    (c) 235
    (d) 268
    (e) 302

    Answer: C

16. Set $k = \sum_{n=1}^{2022} \ln(n)$, and $X = e^k$. X is an integer. When X is written out in base 10, in how many zeroes does it end?

   (a) 404
   (b) 437
   (c) 470
   (d) 503
   (e) 536

   Answer: D

17. A *Collatz sequence* is defined by choosing a positive integer c_1, and then, for all n, setting c_{n+1} equal to $\frac{c_n}{2}$ if c_n is even, and to $3c_n + 1$ if c_n is odd. It is conjectured, but not known, that all Collatz sequences (regardless of the choice of c_1) eventually cycle through the terms 4,2,1. For how many values of c_1 less than 2022 is it the case that c_1 is greater than c_2, c_3, and c_4?

   (a) 503
   (b) 504
   (c) 505
   (d) 506
   (e) 507

   Answer: B

18. The graphs of $y = 2^x$, $y = x^2$, $y = 3^x$, and $y = x^3$ are plotted on the same axes. How many points in the first quadrant lie on two or more of the graphs? (The origin is in the first quadrant, as are both axes.)

   (a) 7
   (b) 8
   (c) 9
   (d) 10
   (e) 11

   Answer: C

19. For any real number x, define the function $f(x)$ to be the minimum value of the three numbers $3x - 4$, $4x - 3$, and $12 - x$. Find the maximum value of $f(x)$.

   (a) 1
   (b) 2
   (c) 4
   (d) 8
   (e) 16

   Answer: D

20. Estimate the value of $\sqrt{29} - \sqrt{21}$ to one decimal place.

   (a) 0.4
   (b) 0.6
   (c) 0.8
   (d) 1.0
   (e) 1.2

   Answer: C

21. Two altitudes of a triangle have length 3 and 4. Find the largest possible integer value for the length of the third altitude.

   (a) 5
   (b) 8
   (c) 11
   (d) 14
   (e) 17

   Answer: C

22. For how many positive integers $n < 2022$ is the number $n^{2022} - 1$ a multiple of 2022?

   (a) 12
   (b) 56
   (c) 84
   (d) 96
   (e) 672

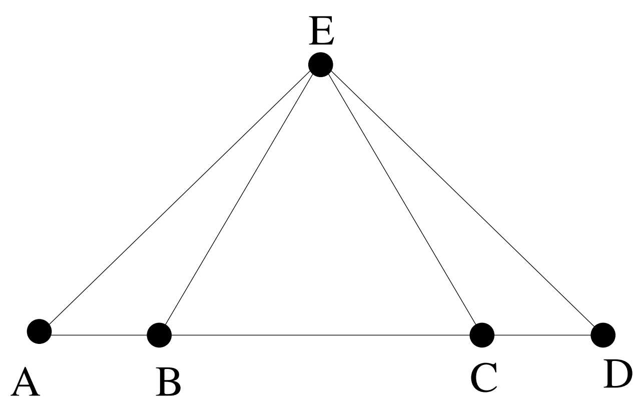
   Answer: A

23. A point P is chosen in the first quadrant and on the line $20x + 21y = 420$ so that, when vertical and horizontal lines are drawn from P to the axes, a square is formed. Find the x-coordinate of point P.

(a) $\frac{29\sqrt{2}}{4}$
(b) $\frac{21}{2}$
(c) $\frac{420}{41}$
(d) $\frac{41}{4}$
(e) $\sqrt{105}$

Answer: C

24. Points A, B, C, D are collinear. If $AB = CD = 6$, $BE = CE = 25$, and the ratio between the perimeters of ADE and BCE is $5 : 4$, find the ratio between the areas of ADE and BCE.



(a) 1.1
(b) 1.2
(c) 1.3
(d) 1.4
(e) 1.5

Answer: D

25. The number $27,000,001$ has exactly 4 prime factors. What is the sum of those prime factors?

(a) 648

(b) 650

(c) 652

(d) 654

(e) 656

Answer: C