Team Number:

1. Goldbach's conjecture asserts that every even number greater than 4 can be written as the sum of two odd primes. It is not known whether this statement is true or false.

   On the other hand, it is known that every sufficiently large even number can be written as a sum of two odd *composite* numbers.

   How many positive even numbers cannot be written as a sum of two positive odd composite numbers?

   [0 is not positive, and 1 is neither prime nor composite.]

Team Number:

2. Remember that a *palindrome* is a number (or word) that reads the same backwards and forwards. For example, 353 and 2112 are palindromes.

   Frank begins driving his car, which has a six-digit odometer (which tells him how many miles the car has driven). He notices that the last four digits of the odometer form a palindrome. After driving one mile, he notices that the last five digits of the odometer form a palindrome. He drives another mile and notices that the middle four digits of the odometer are a palindrome. Finally, he drives one more mile, and all six digits of the odometer form a palindrome. There are two possible initial odometer readings that could have given Frank these observations. Find one of them.

Team Number:

3. The equation

$$2023 = 9^3 + 8^3 + 7^3 + 6^3 + 5^3 + 4^3 + 3^3 + 2^3 + 1^3$$

is false, but can be made true by changing some of the plus signs to minus signs. How many signs need to be changed?

Team Number:

4. King Arthur's round table has 2023 chairs, arranged in a circle. K of Arthur's knights sit around the table in such a way that no two knights are in adjacent chairs, but a $(K+1)^{\text{st}}$ knight would not be able to join without sitting next to somebody.

   How many values of K are possible?

Team Number:

5. In the figure (not drawn to scale), triangles ABC, ACF, CDF and DEF are all right triangles (with right angles at B, C, C, and F, respectively).

   If all sides of all four triangles have integer length, and $AB = 3$, there are several possibilities for the length of DE. Find two of them.