HW 11, Due Friday April 20

1) Rudin page 332 exercise 8

2) Rudin page 332 exercise 11. Here consider the set $\mathcal{L}(\mu)$ (what I called $L^1(\mu)$ in my notes) up to equality almost everywhere. So really consider the set of equivalence classes where two elements are the same if they are equal almost everywhere.

3) Rudin page 332 exercise 16. Feel free to use the following version of Riemann-Lebesgue lemma (though we haven't proved it): For every measurable $X \subset \mathbb{R}$ with $m(X) < \infty$, we have

$$\lim_{n\to\infty} \int_X \sin nx\, dx = \lim_{n\to\infty} \int_X \cos nx\, dx = 0.$$

Note that the exericise shows that no subsequence of $\{\sin nx\}$ can converge, and in fact, no subsequence can converge on any set of positive measure.

4) Let $\{f_n\}$ be a sequence of measurable functions converging uniformly to 0, show that

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} \frac{f_n(x)}{1+x^2}\, dx = 0.$$

5) Show that

$$\lim_{b\to\infty} \int_0^b \frac{\sin(x)}{x}\, dx$$

exists, but

$$\lim_{b\to\infty} \int_0^b \left|\frac{\sin(x)}{x}\right|\, dx = \infty.$$

Conclude that $\frac{\sin(x)}{x}$ is not in $L^1(\mathbb{R})$.