HW 8, Due Friday Mar 23

1) If $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.

2) a) Prove that m^* is translation invariant. That is, given a set A and $x \in \mathbb{R}$, and

$$A + x = \{y + x \in \mathbb{R} : y \in A\}$$

then

$$m^*(A) = m^*(A + x).$$

b) Show that if E is Lebesgue measurable, then given any $x \in \mathbb{R}$, the set $E + x$ is Lebesgue measurable.

3) If $\{E_j\}$ are all Lebesgue measurable sets such that $E_1 \supset E_2 \supset E_3 \supset \cdots$ and let $E = \cap_{j=1}^{\infty} E_j$. Suppose that $m(E_1) < \infty$ then

$$\lim_{j\to\infty} m(E_j) = m(E).$$

4) Find an example of sets E_j with $E_1 \supset E_2 \supset E_3 \supset \cdots$ but such that $m(E_j) = \infty$ for all j, and $\cap_{j=1}^{\infty} E_j = \emptyset$. So the hypothesis that E_1 is of finite measure in exercise 3) is necessary.

Note that a previous version of the exercise just said $m(E_1) = \infty$, that was a typo, it should have been $m(E_j) = \infty$ as it now says.

5) Prove that if $\{E_j\}$ is a countable collection of pairwise disjoint Lebesgue measurable sets and $A \subset \mathbb{R}$ is any set then

$$m^*\left(A \cap \bigcup_{j=1}^{\infty} E_j\right) = \sum_{j=1}^{\infty} m^*(A \cap E_j).$$