Drawing knots using computers

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University of Texas at Austin

UnKnot 2009 Conference
July 16th 2009
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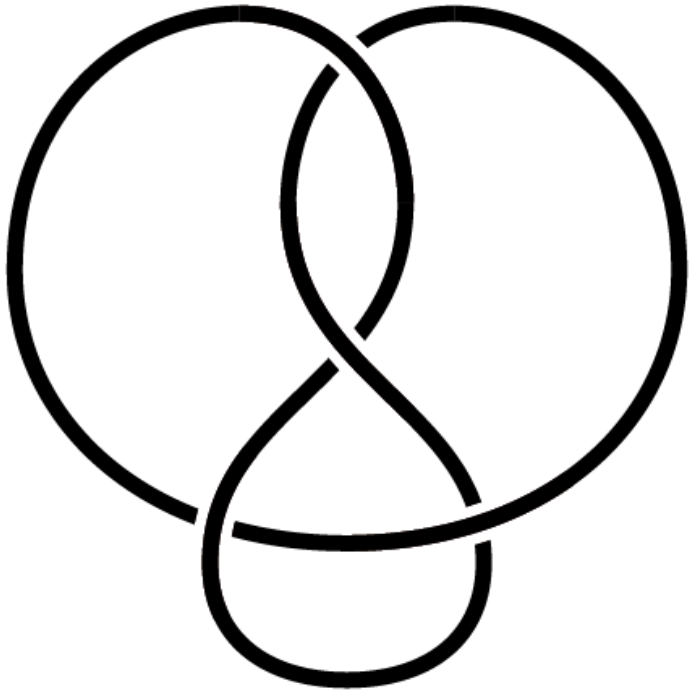
2D: Vector graphics programs



- Adobe Illustrator ($200 for student version)
- Inkscape (free)
- KnotsBag (free demo, 10 Euros for full version)

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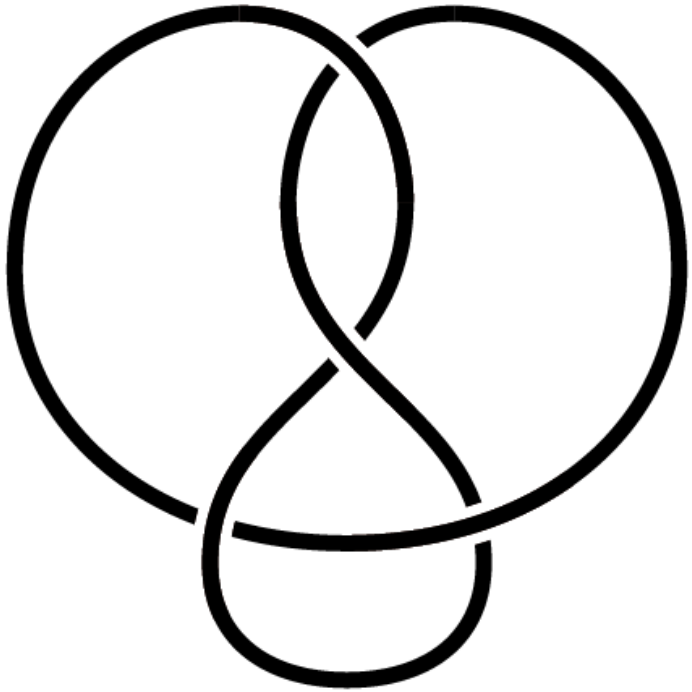

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Bézier curves

Cubic Bézier curves are used in many applications to get smooth curves with specified start and end points, and specified start and end "velocities".

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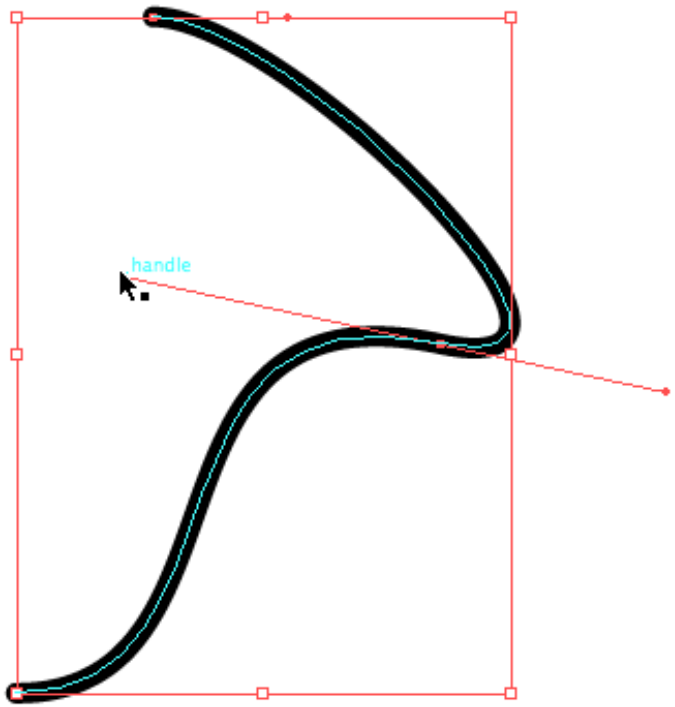
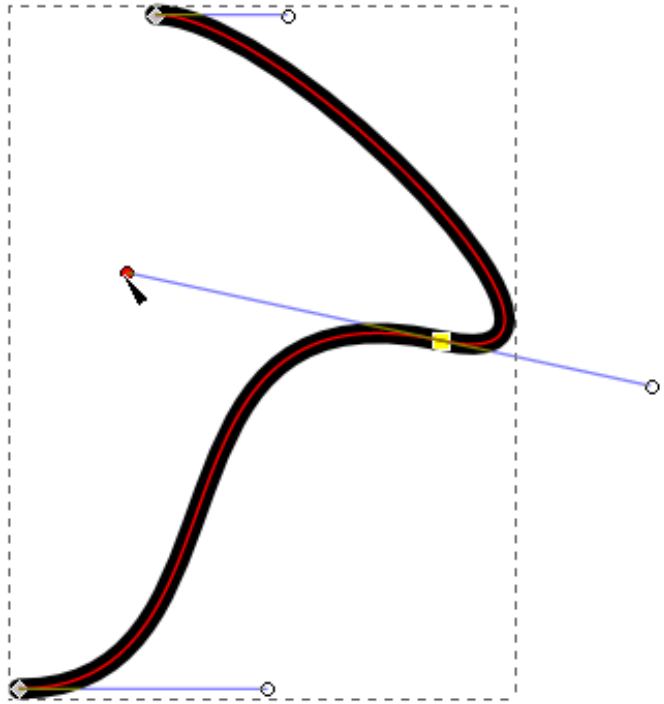
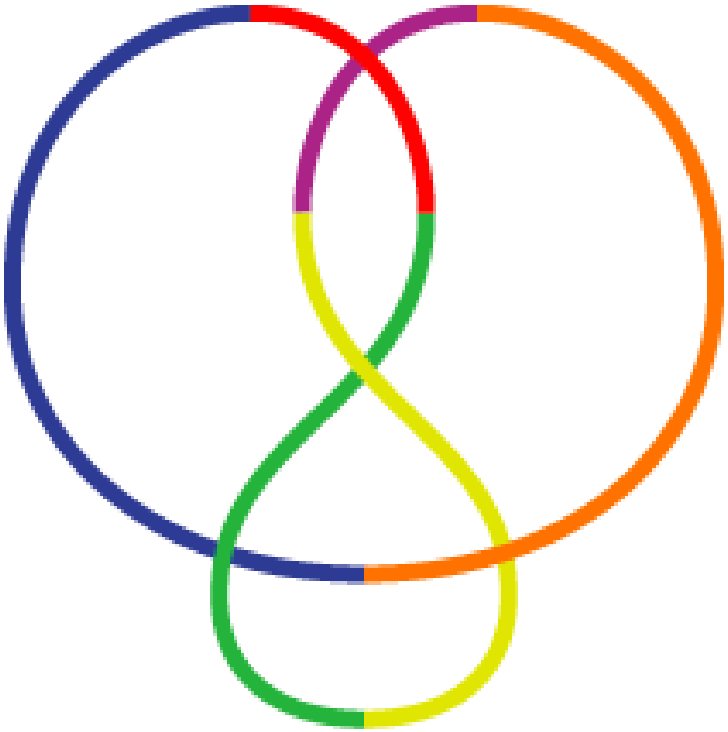
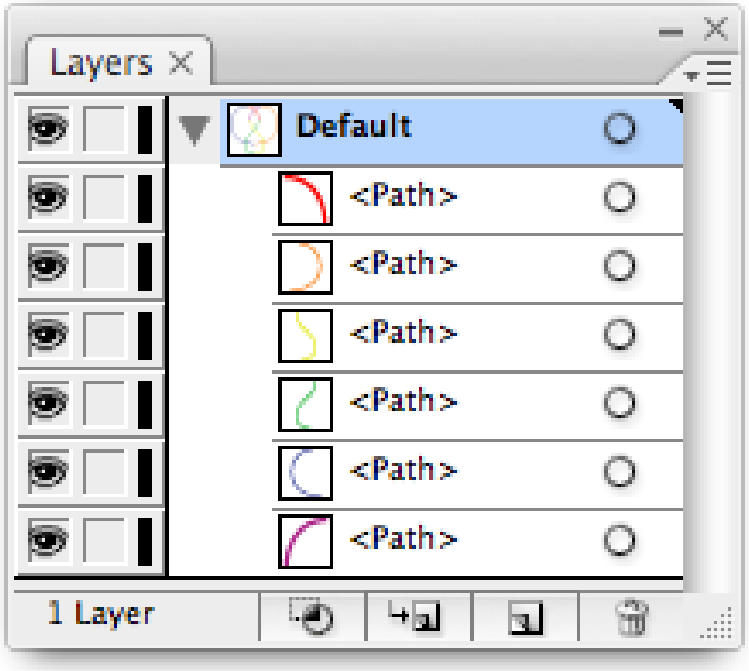


Inkscape:



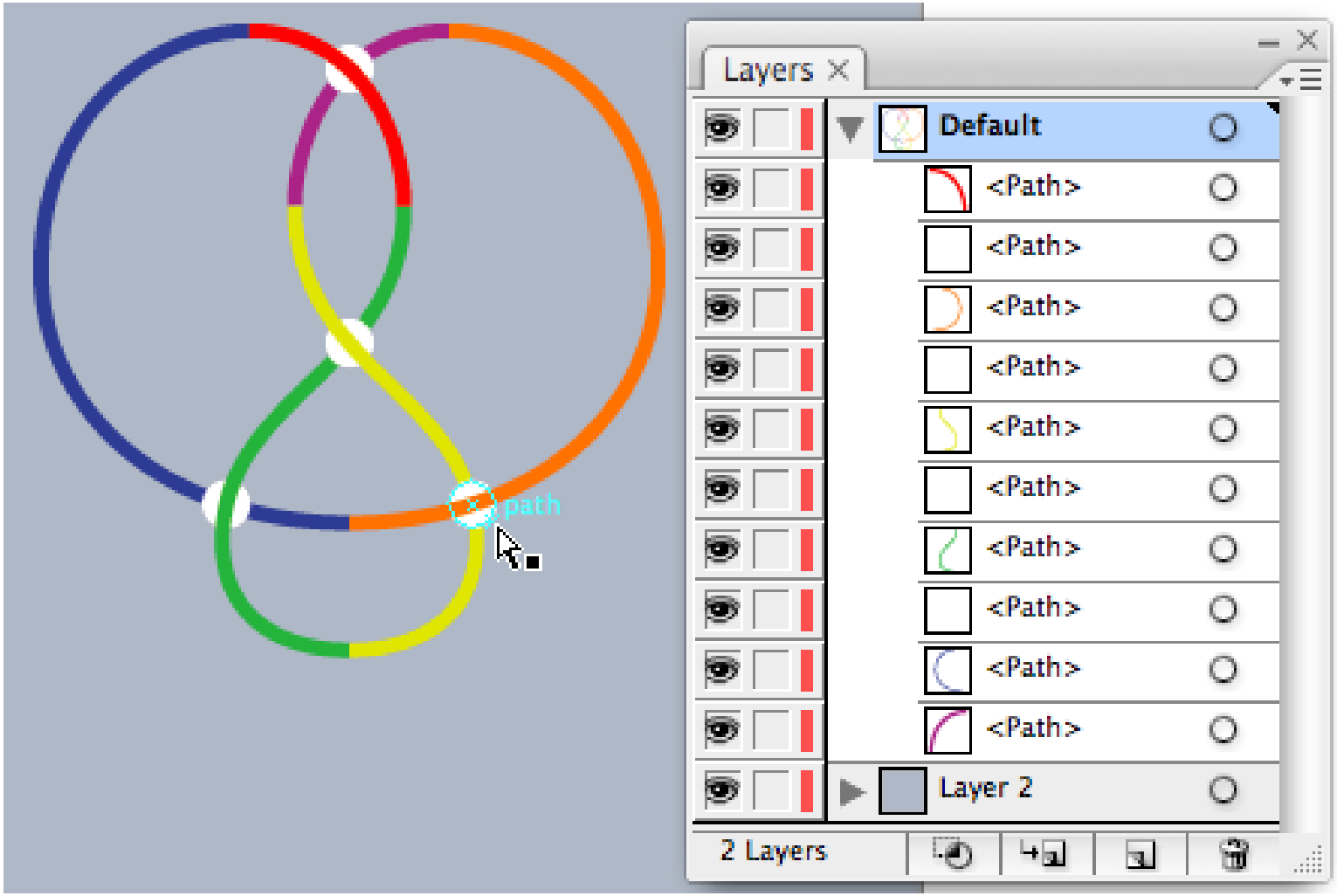
Crossings



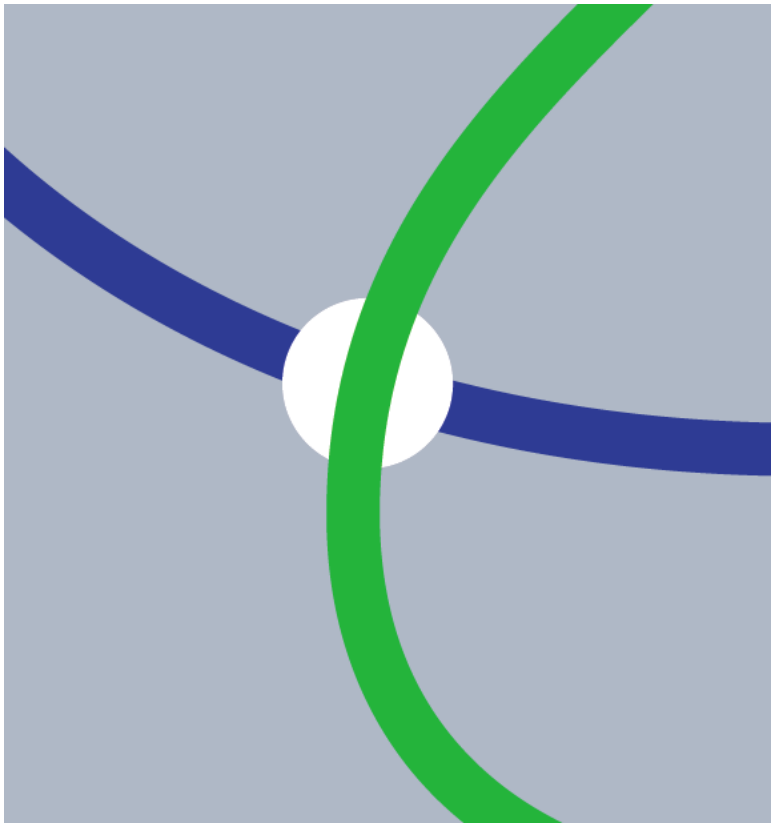


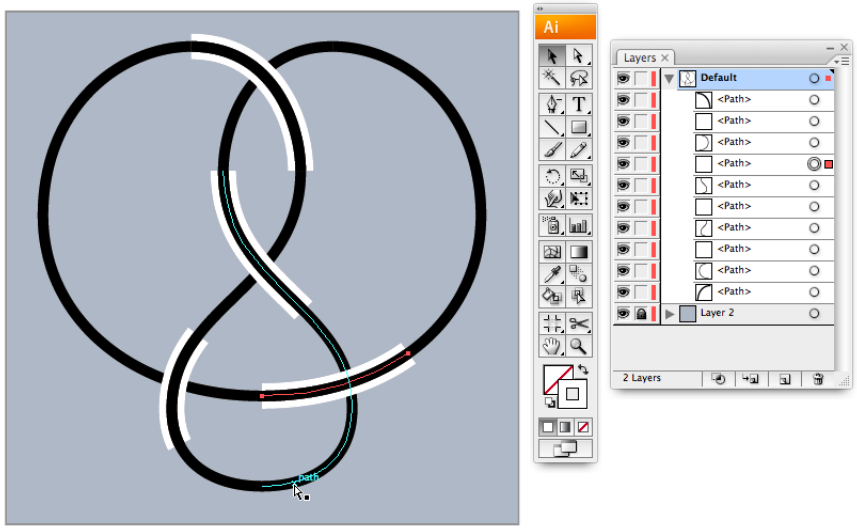
Arrange the heights of the curves so that the crossings are correct.

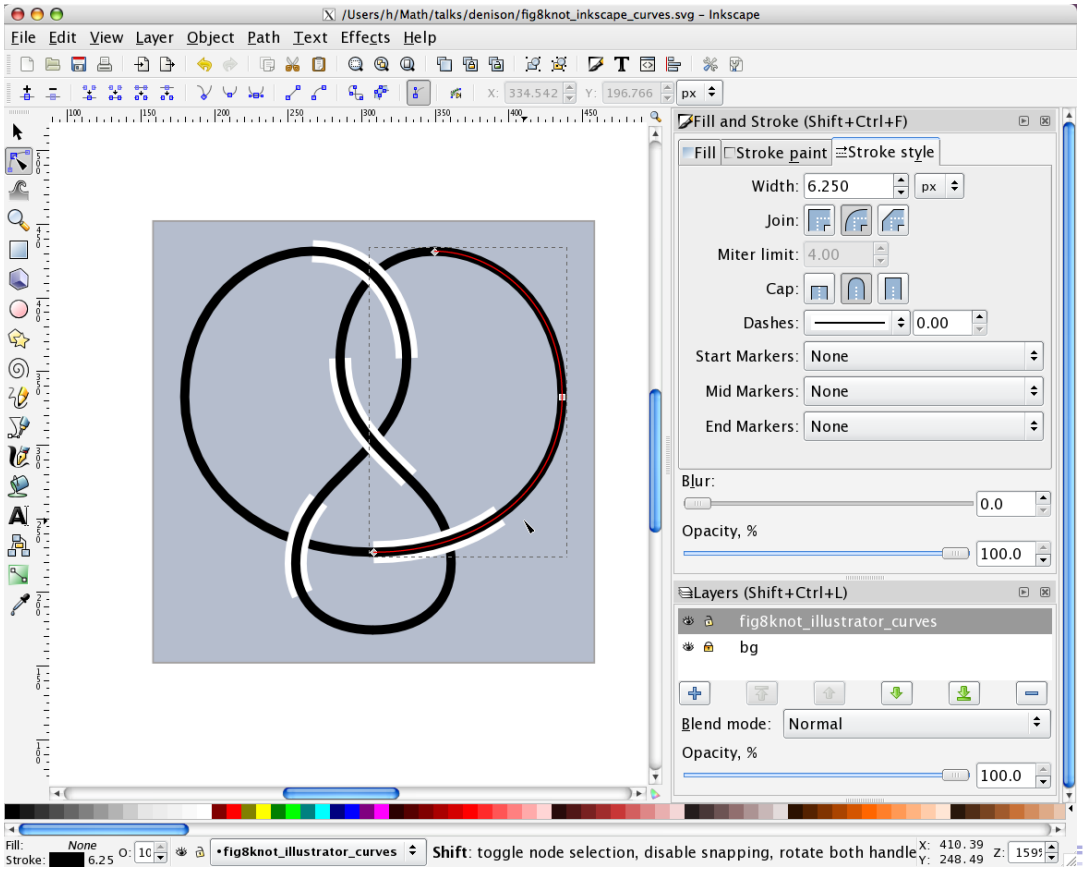
To get the usual "gap" style for a crossing, the usual trick is to insert a white disk between overlapping objects:



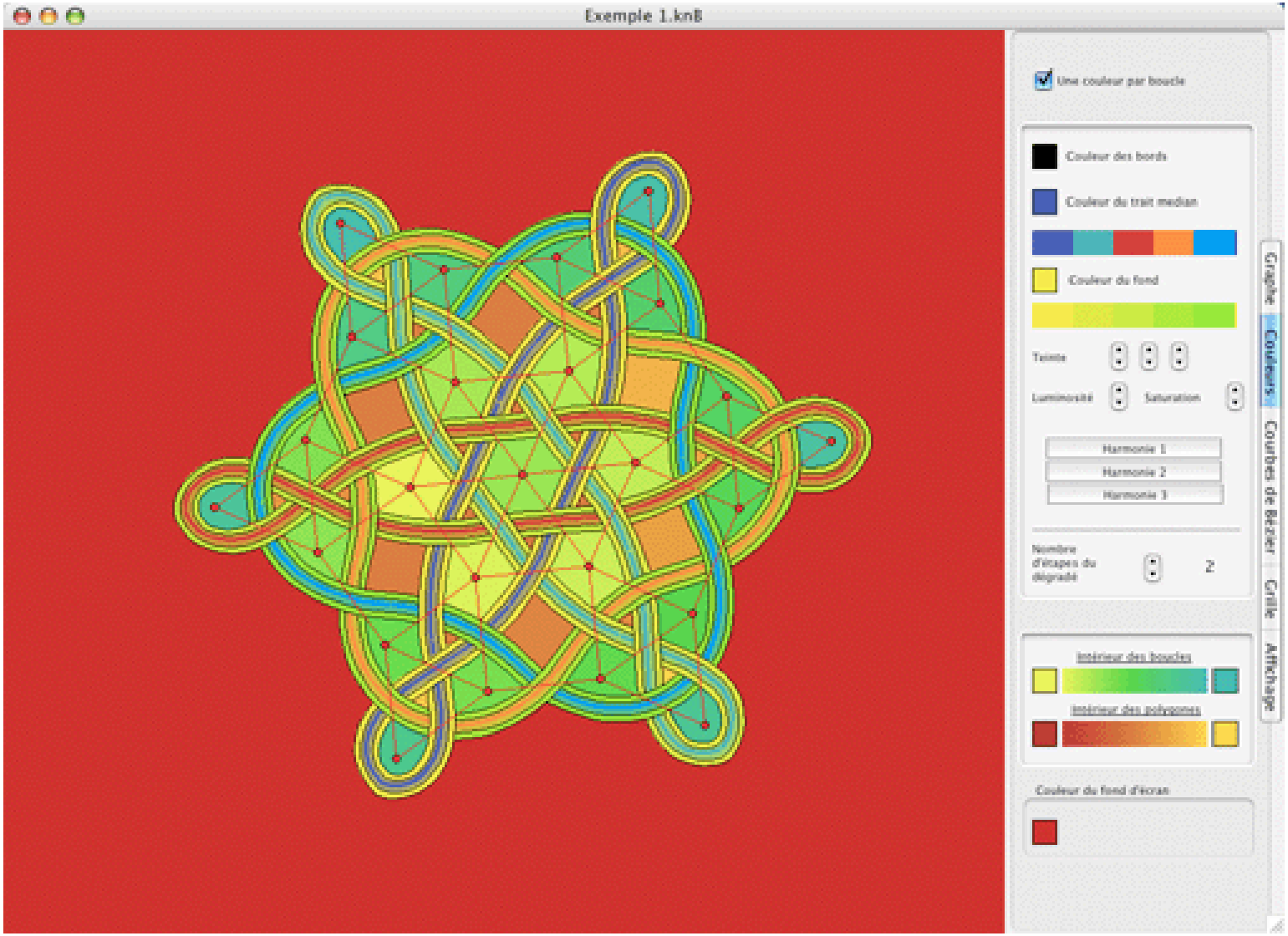
Sometimes it is better to use a copy of the upper curve instead of a disk.







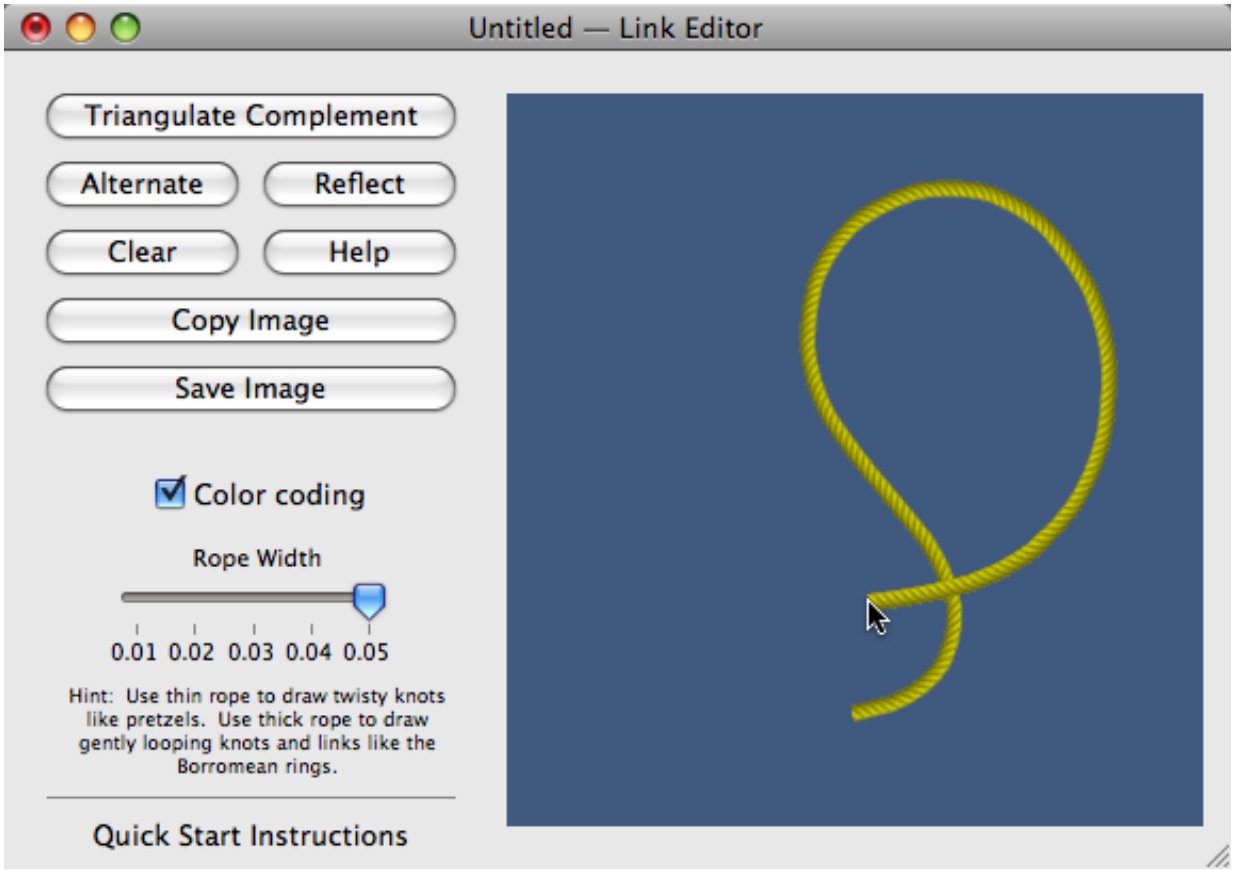
KnotsBag



SnapPea

SnapPea is a free program by Jeffrey Weeks for computational study of 3-manifolds.

One of the ways to specify a manifold is as a knot or link complement.



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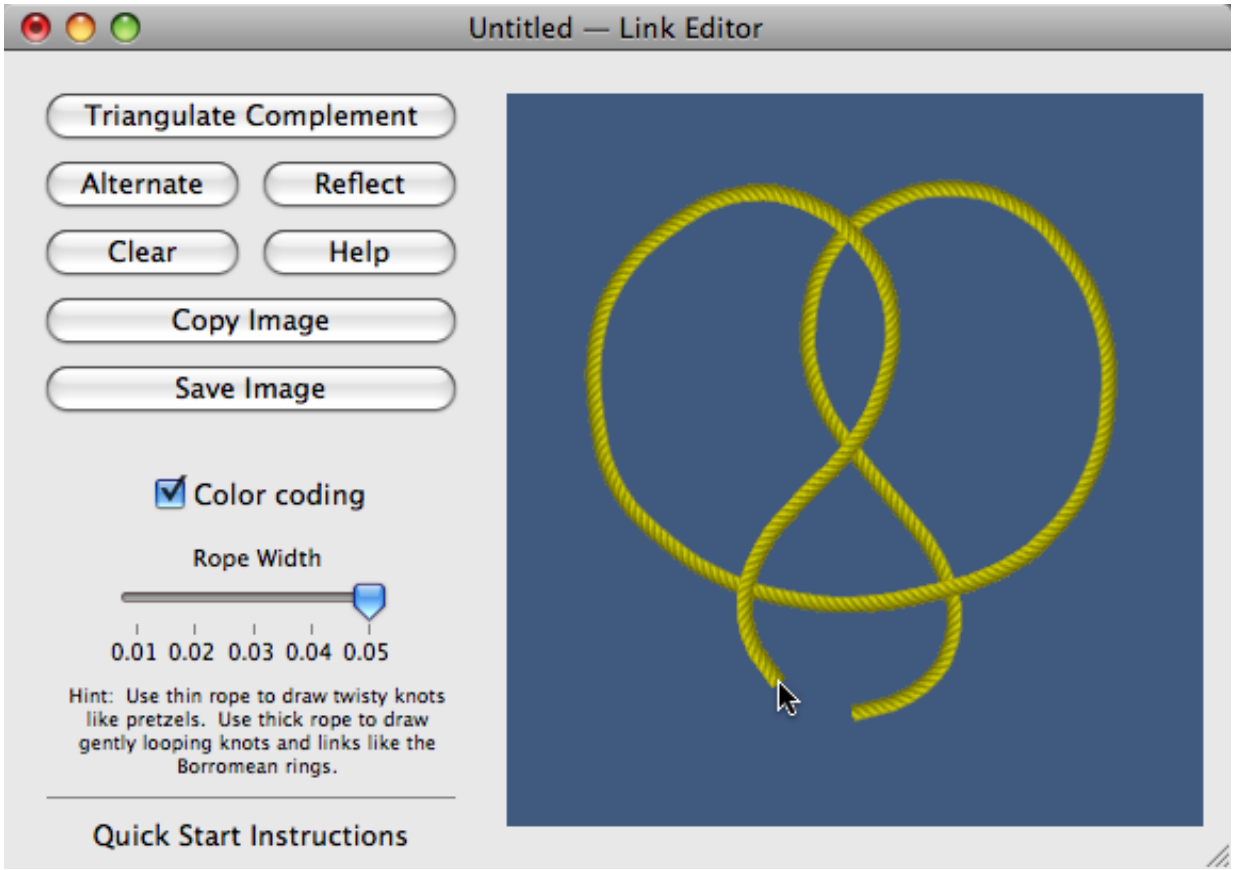

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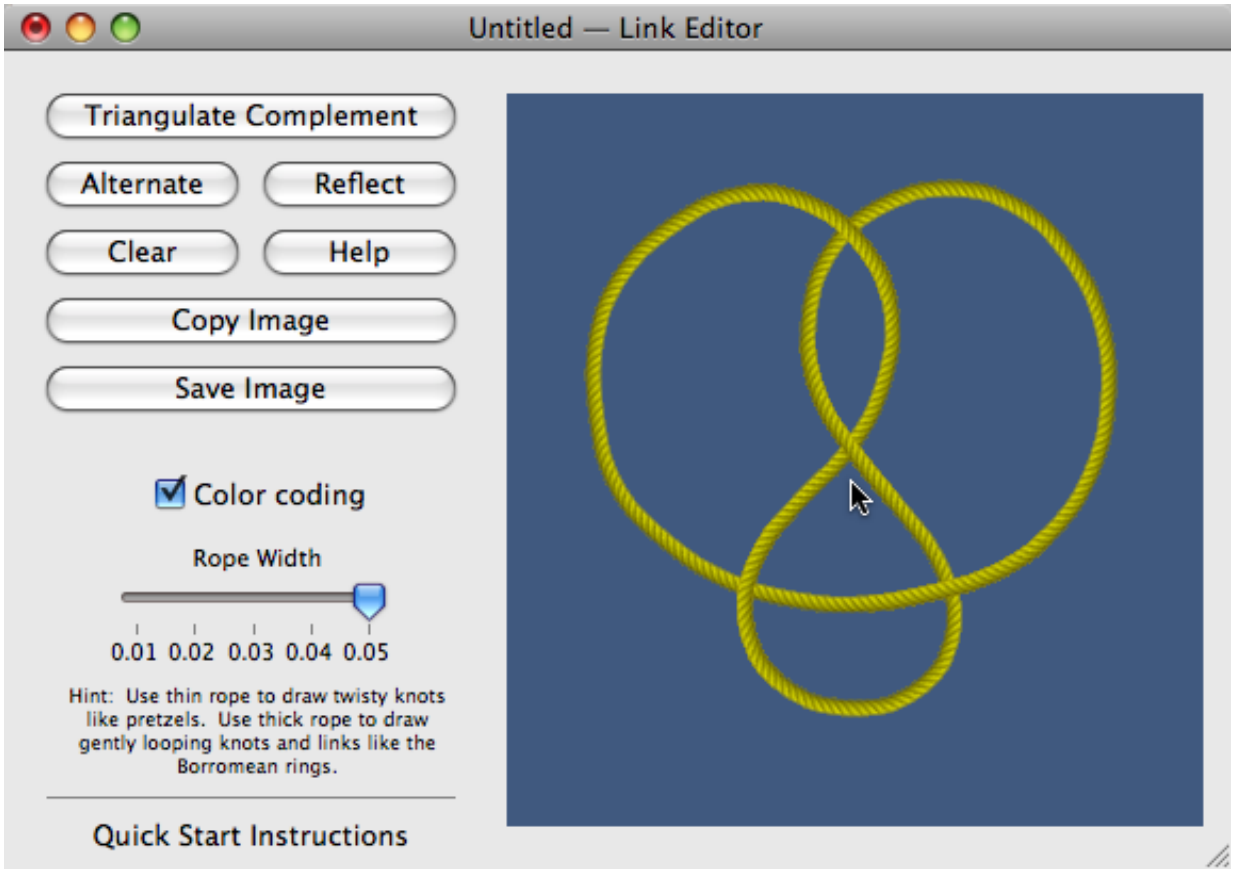
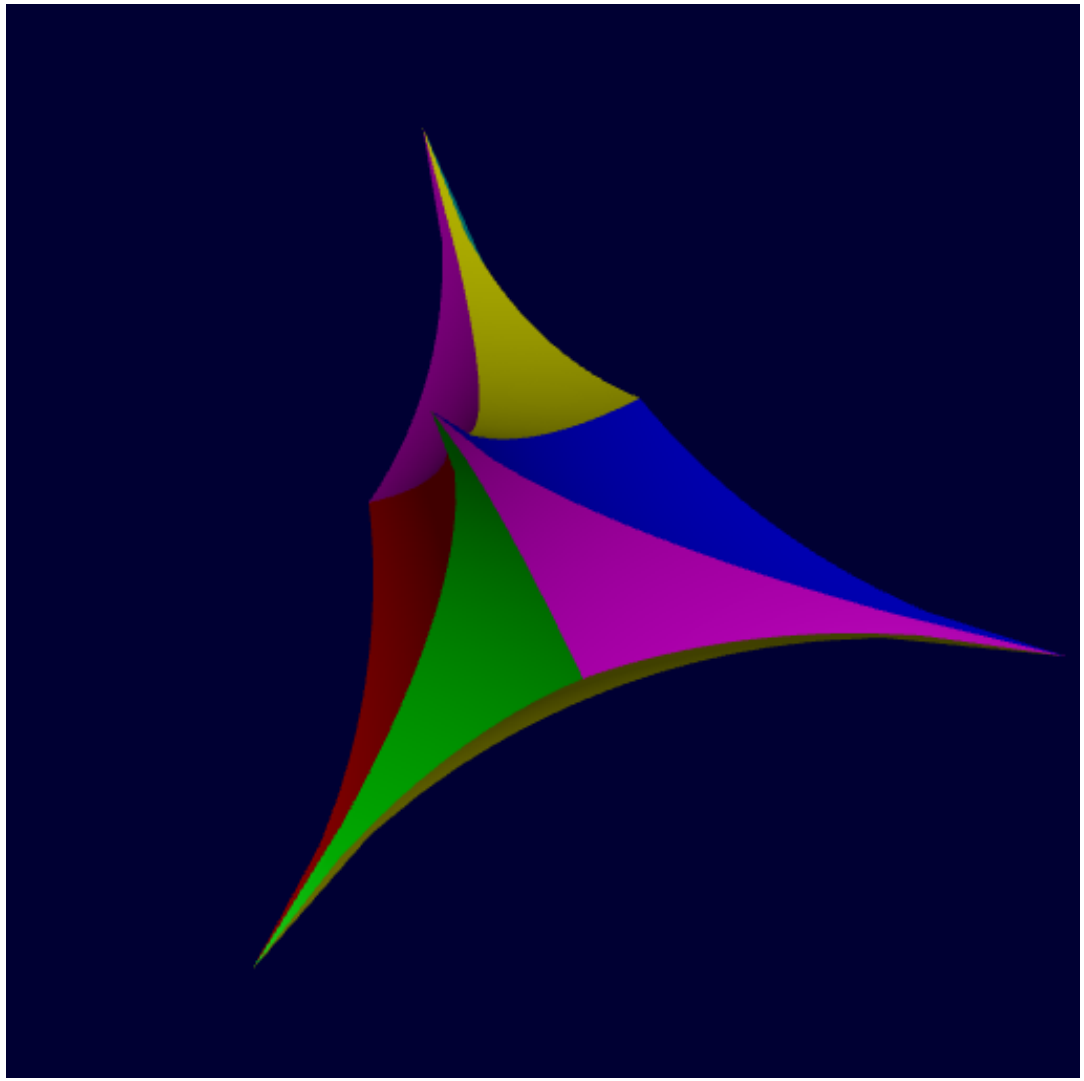


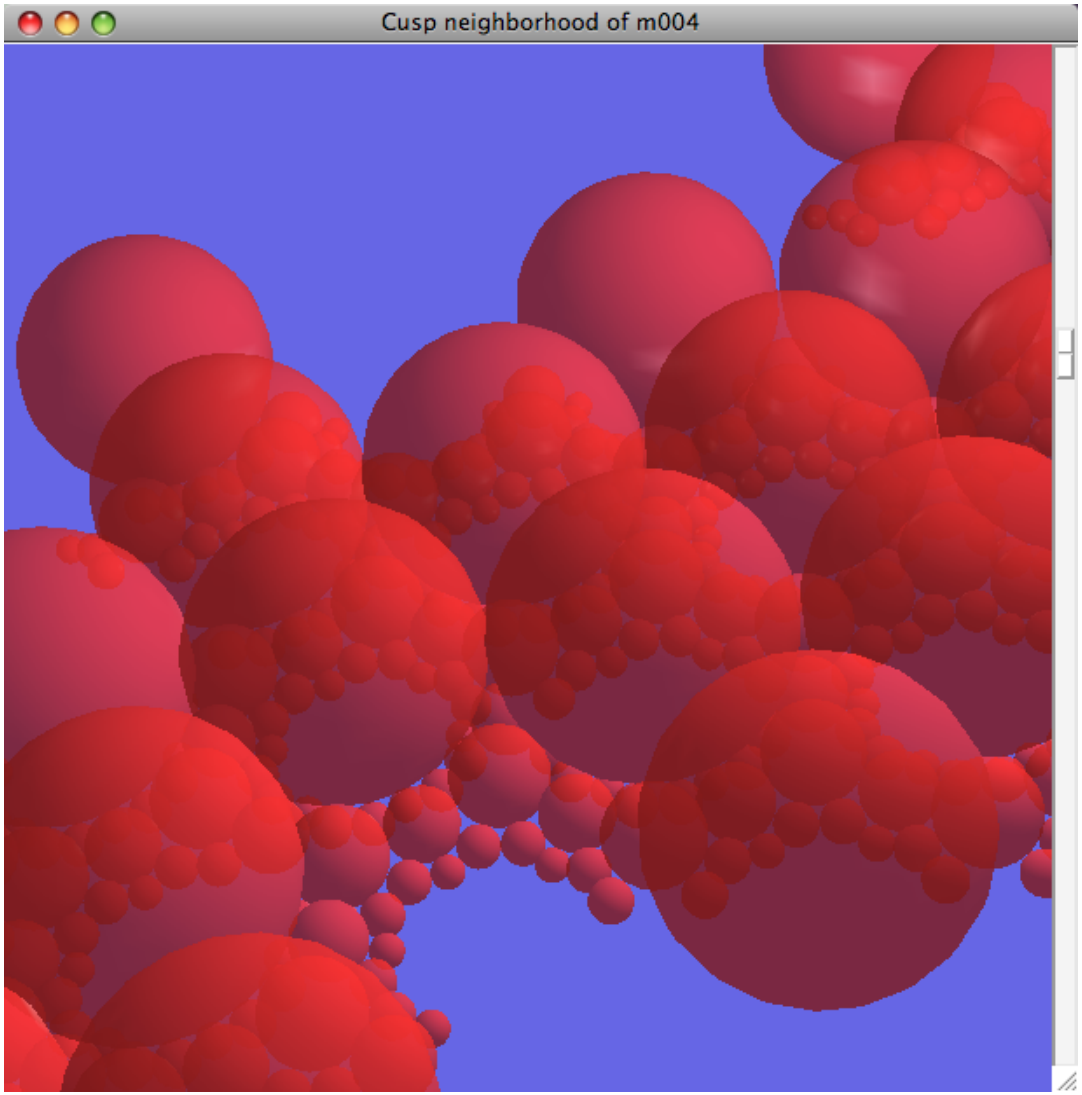
Once we have a manifold specified, we can get all sorts of interesting information out.

Dirichlet domain:



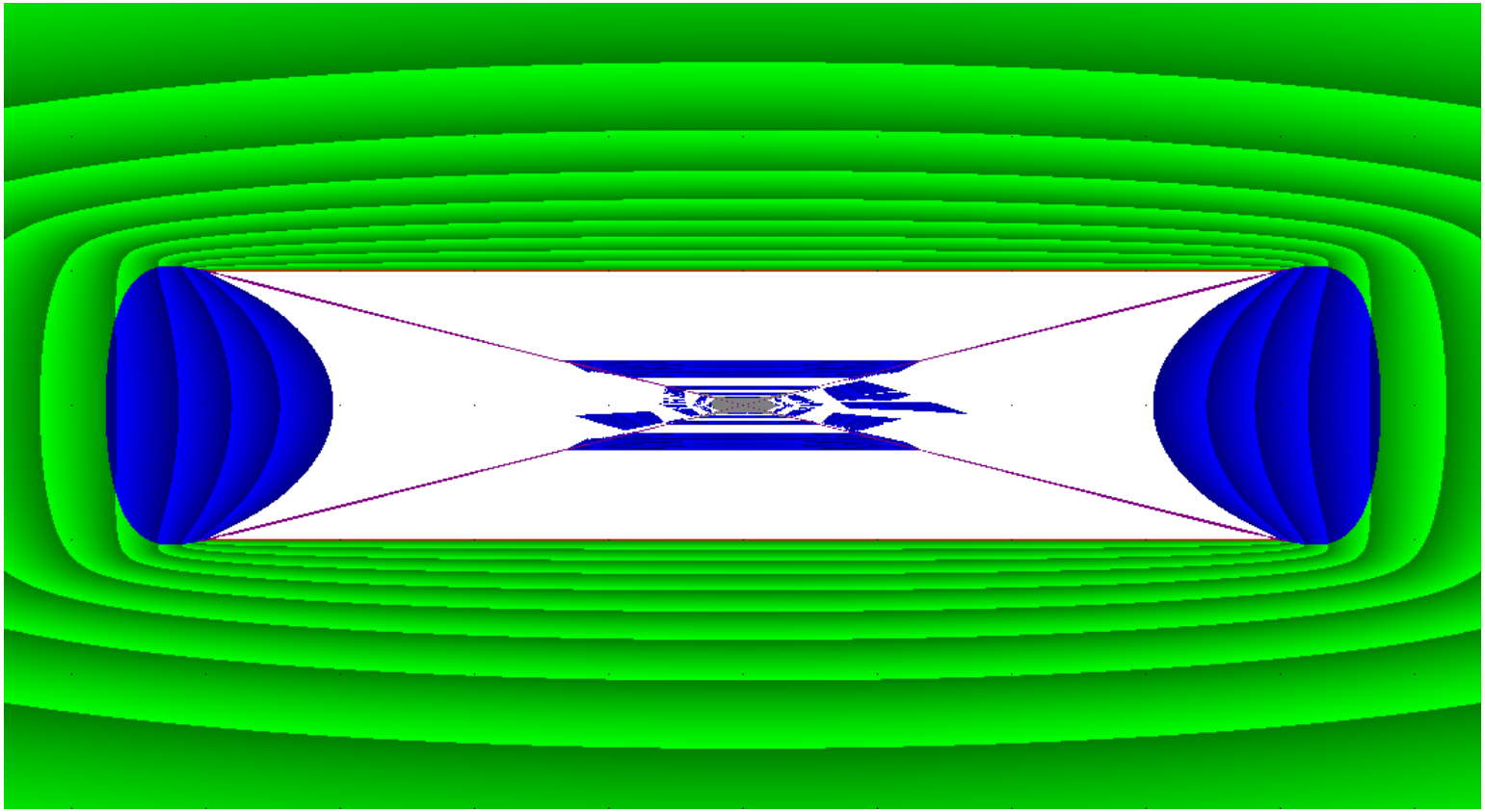
(From SnapPy by Marc Culler and Nathan Dunfield)

Cusp Neighbourhoods:

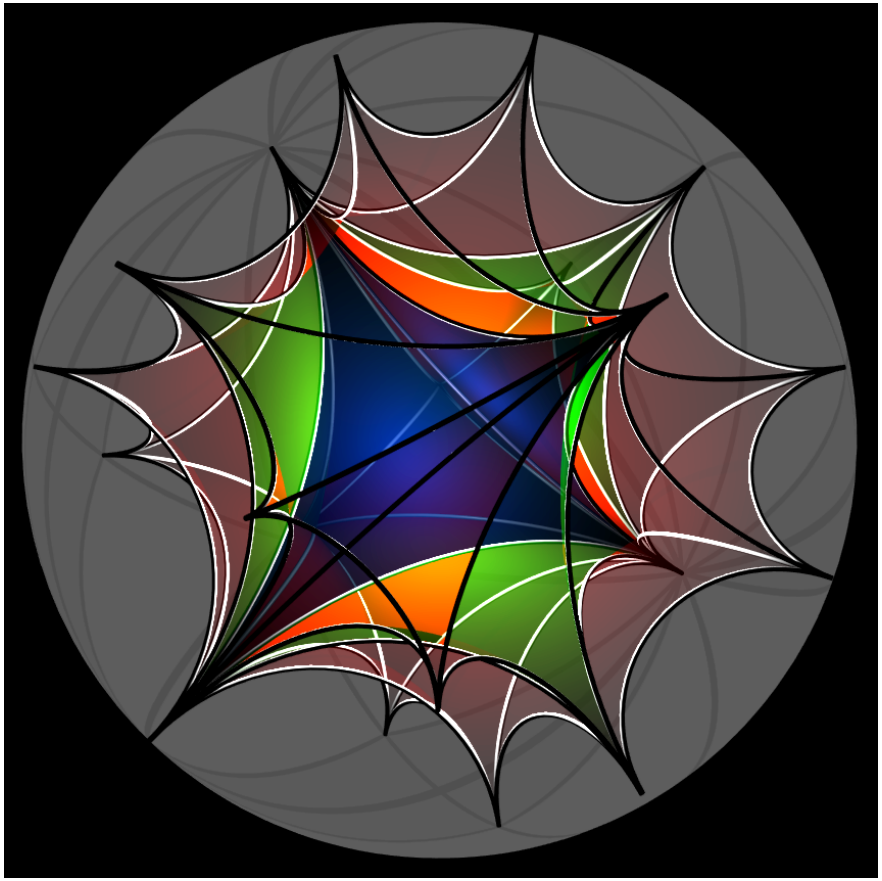


(Using SnapPeaPython by Marc Culler and Nathan Dunfield)

Dehn Surgery space:



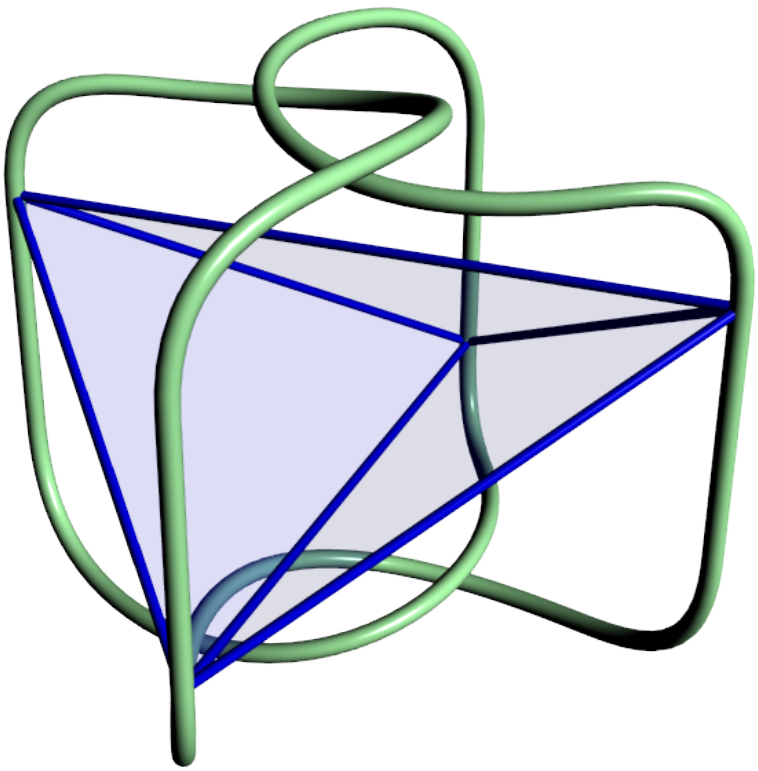
3D

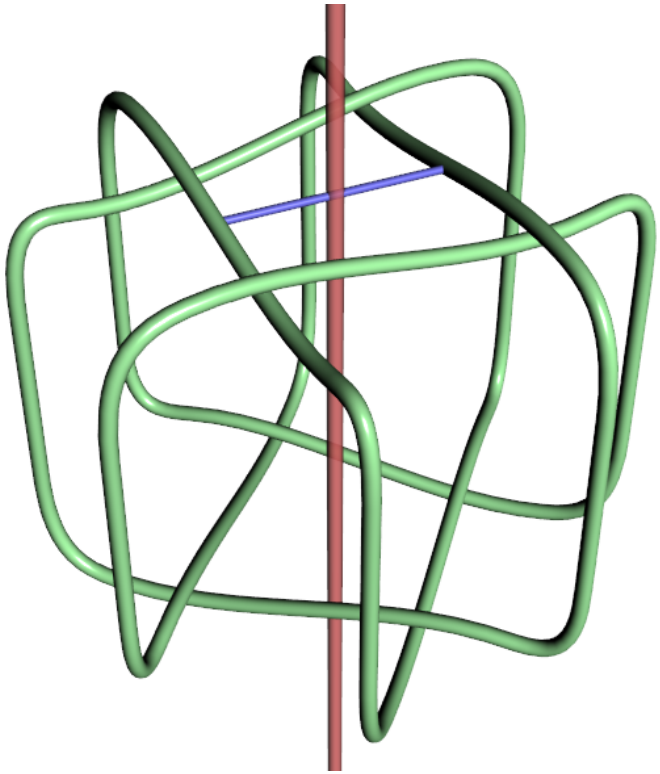


Why bother working in 3D?

- Beautiful renderings for talks, papers, T-shirts, etc.
- Get a better understanding of the object.
- Build the object first, choose the viewpoint second.
- 3D printing.

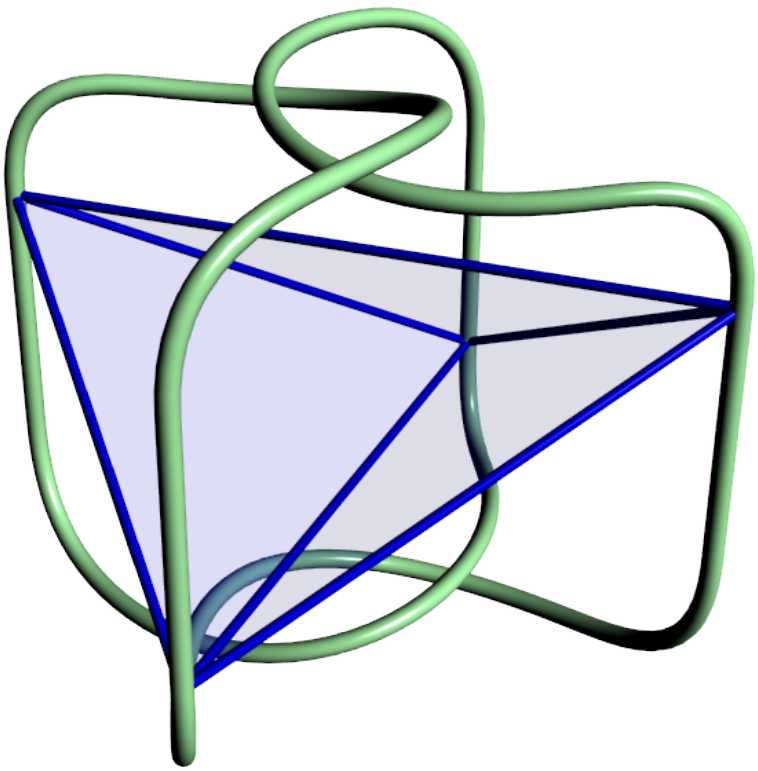
CAD programs

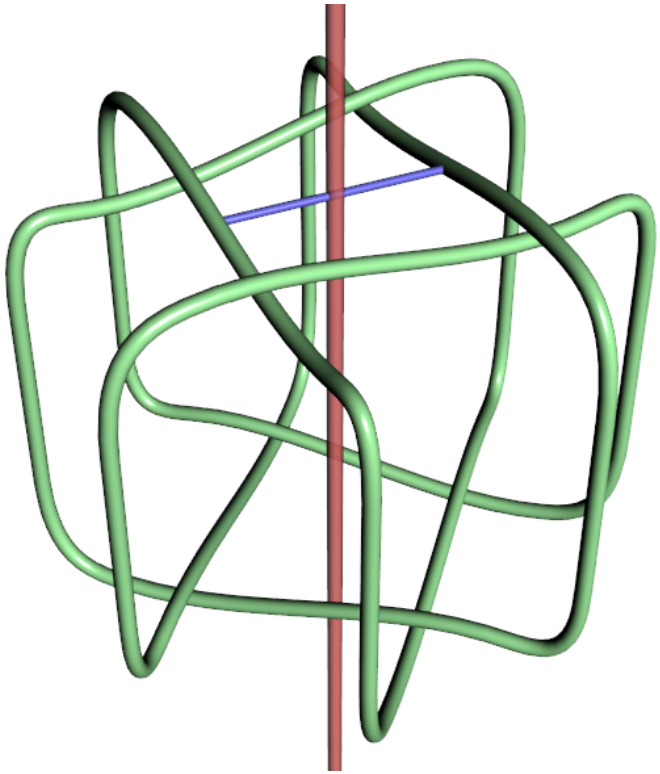




- Rhinoceros 3d ($140 ∼ $200 for students and teachers)
- Blender (free)
- Maya, 3ds Max, etc. (If you have to ask...)

CAD programs





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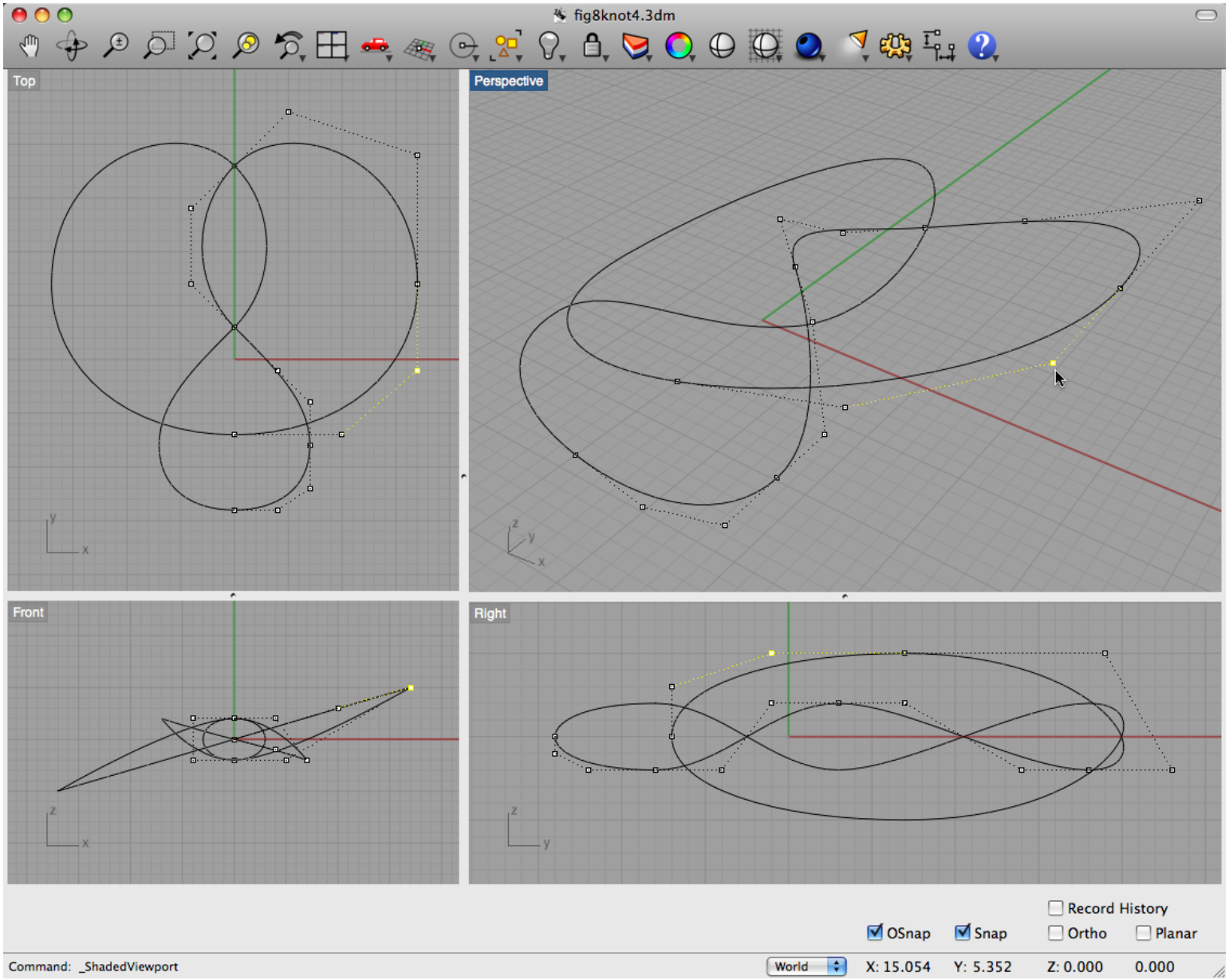
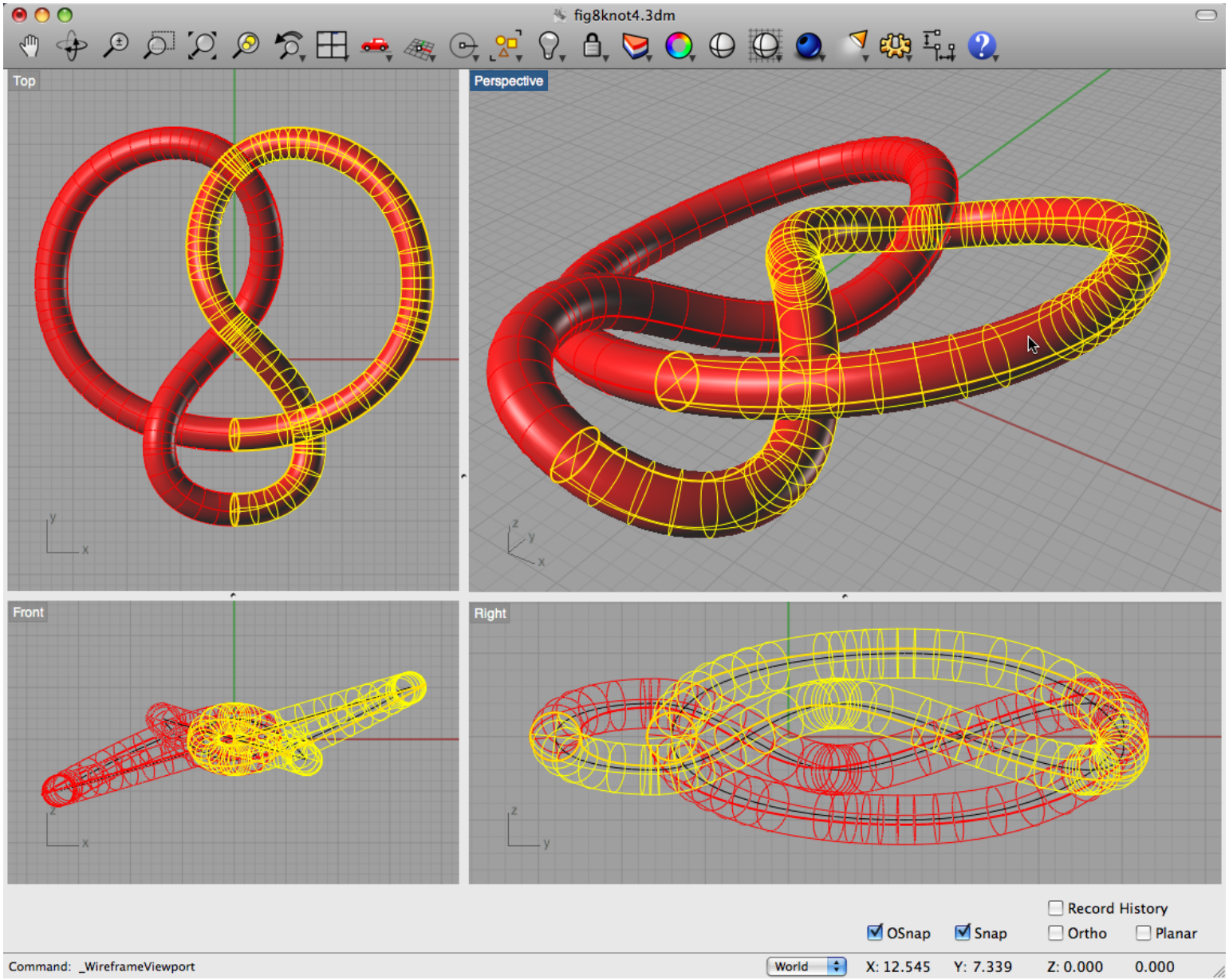
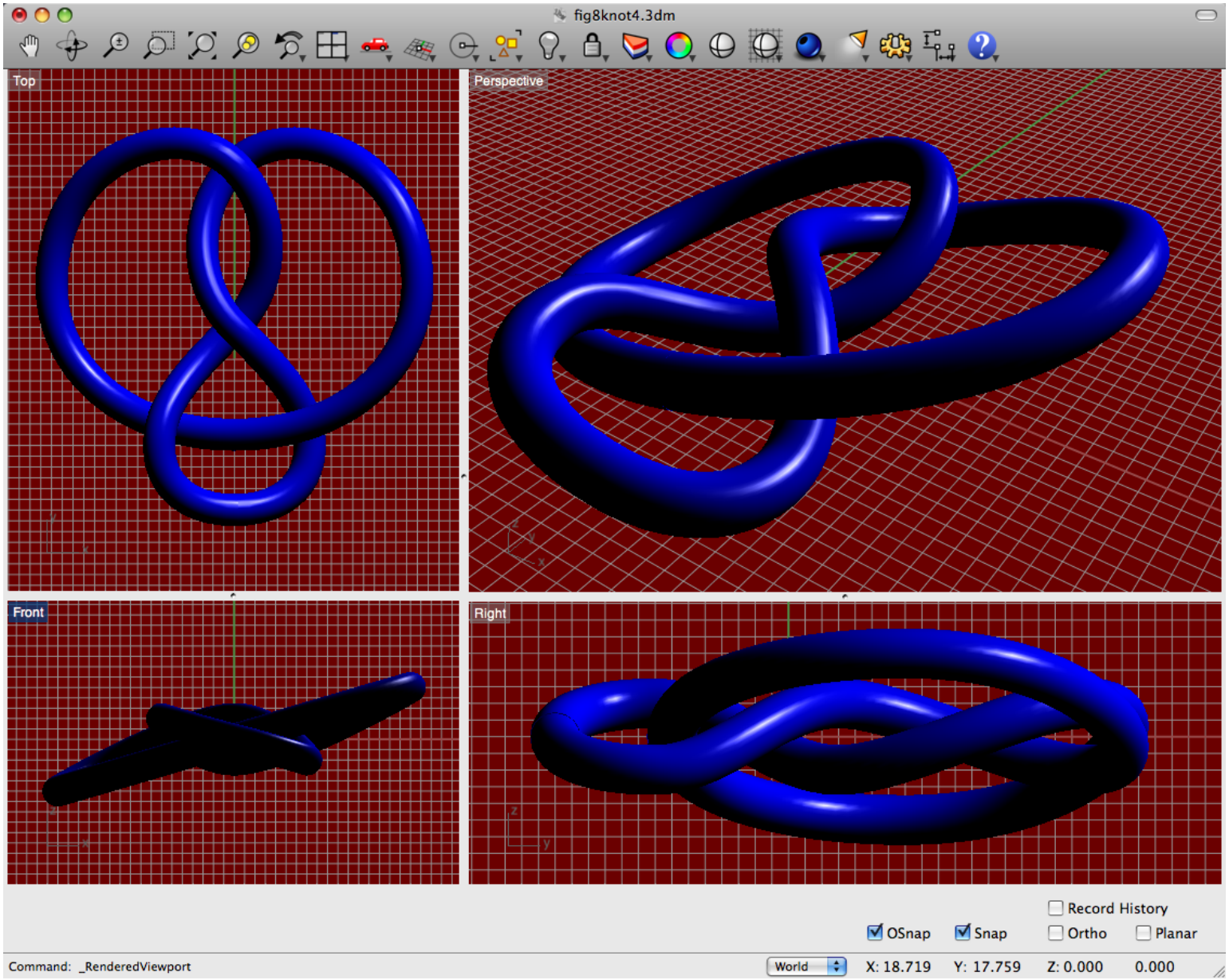
Bézier curves



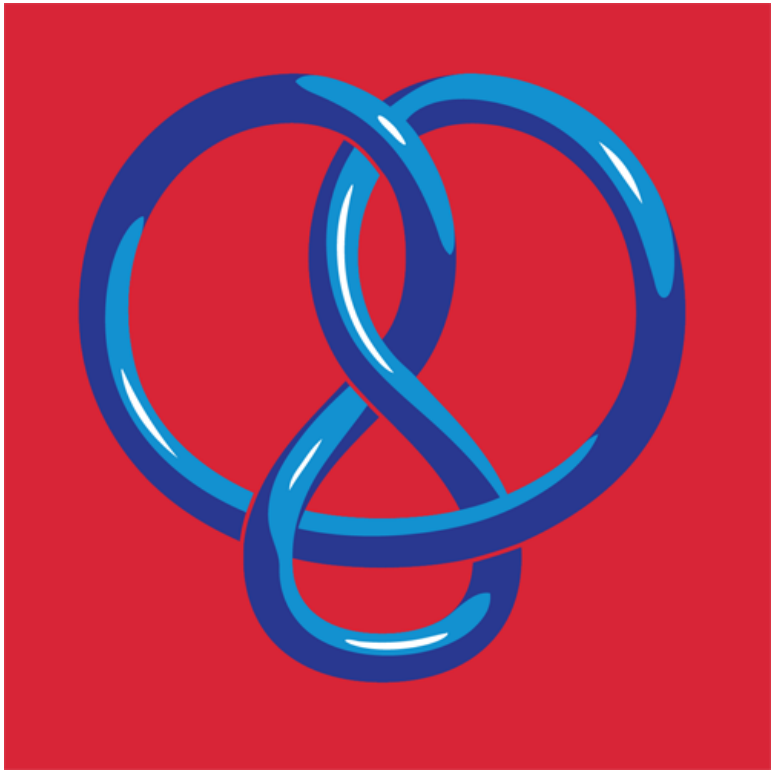
Pipes



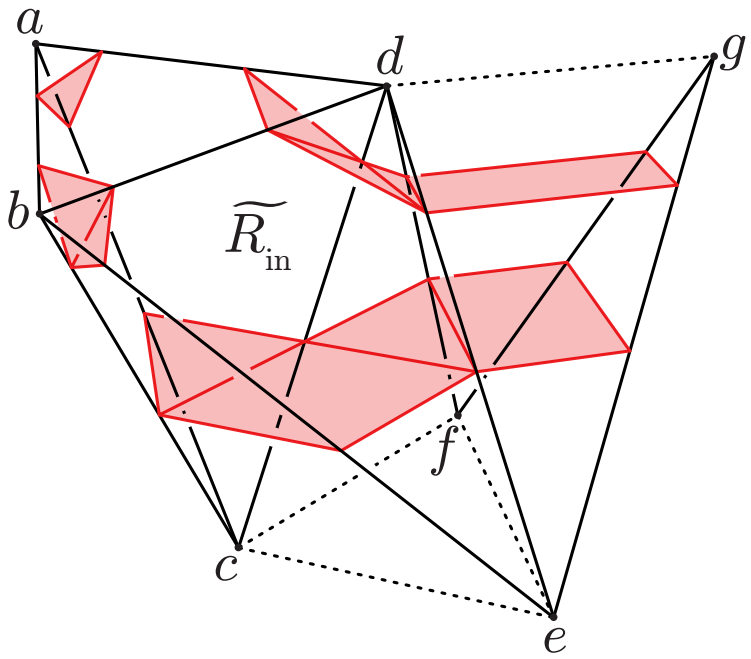
Render



Next...



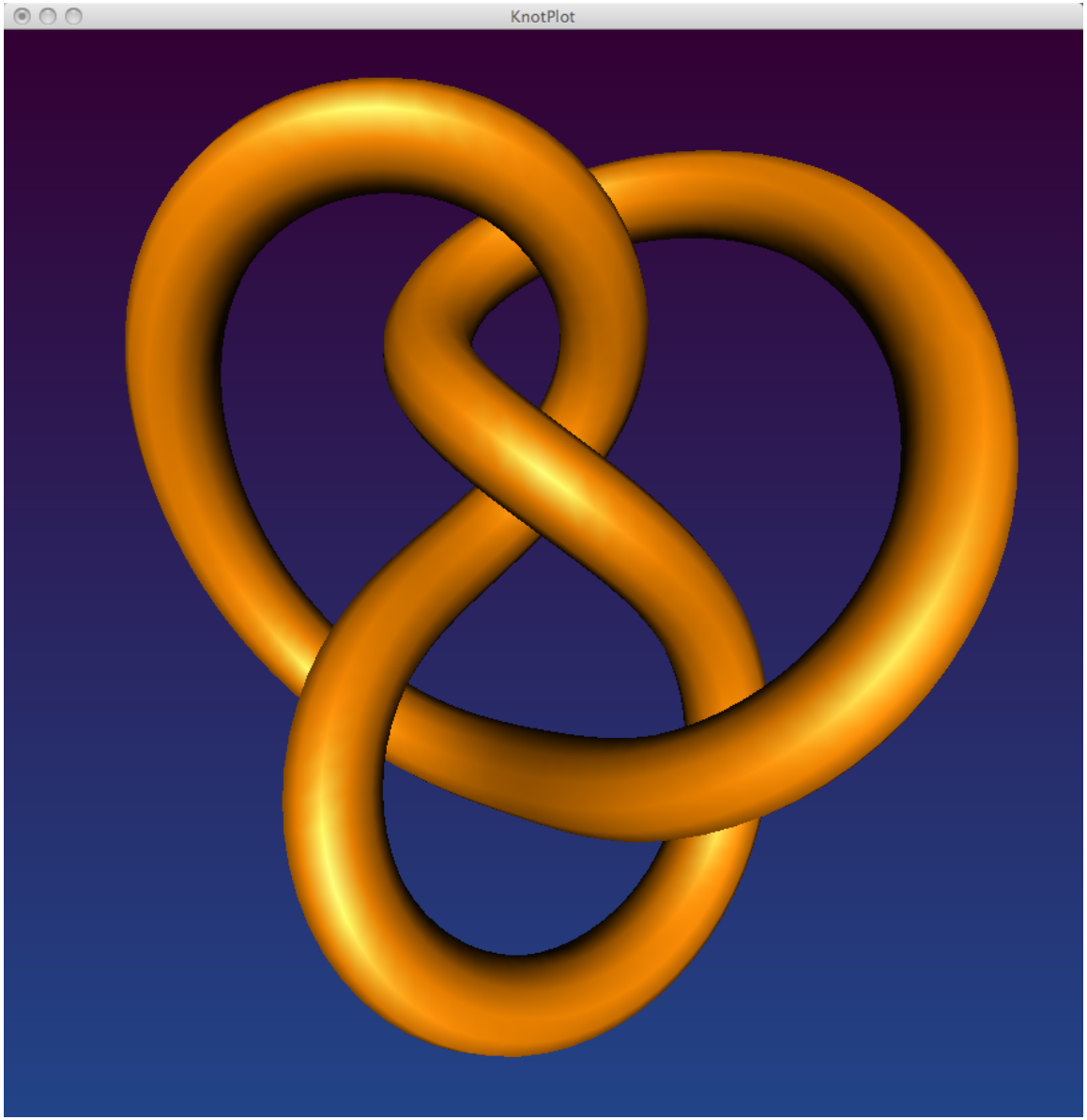
Take the render as a pixels image, modify it in a pixel editing program (e.g. Photoshop, GIMP, etc.), etc.

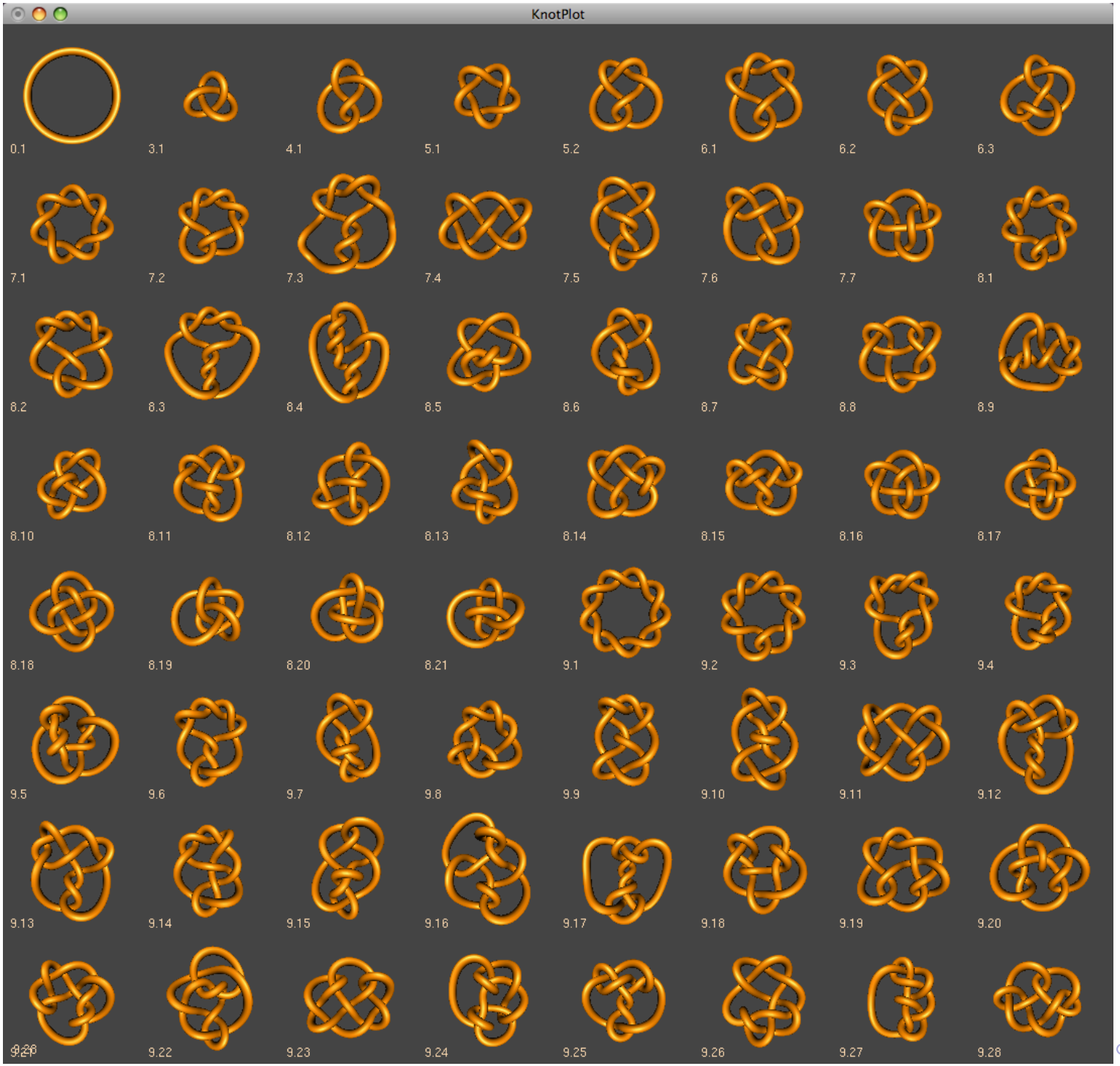


...or export a particular viewpoint or your 3D model as a 2D vector graphics image, modify it in Illustrator (or Inkscape, etc.) and use in your paper, etc.

Knotplot

(free to evaluate, $24.99 CAD for full version)




KnotPlot
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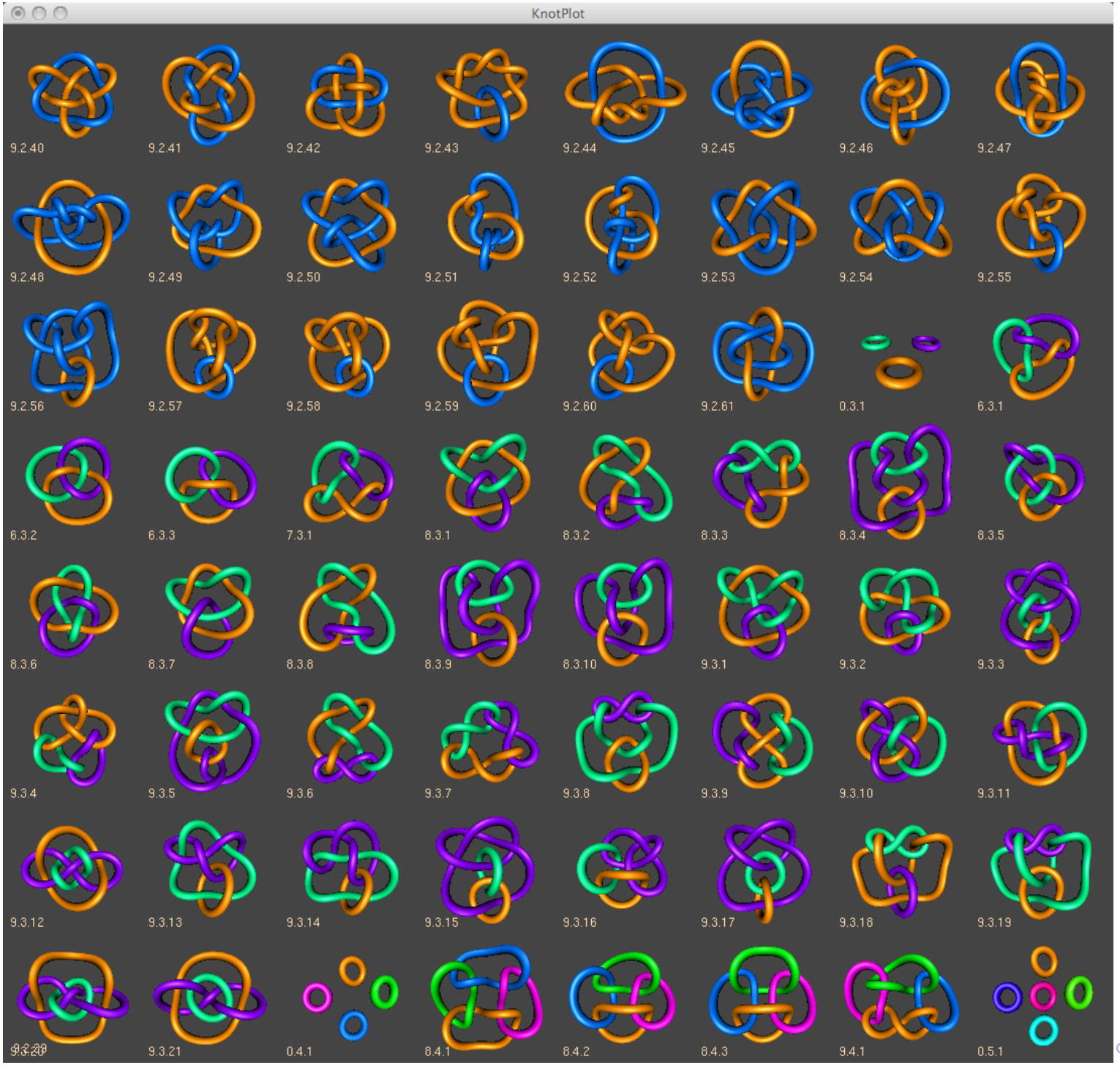
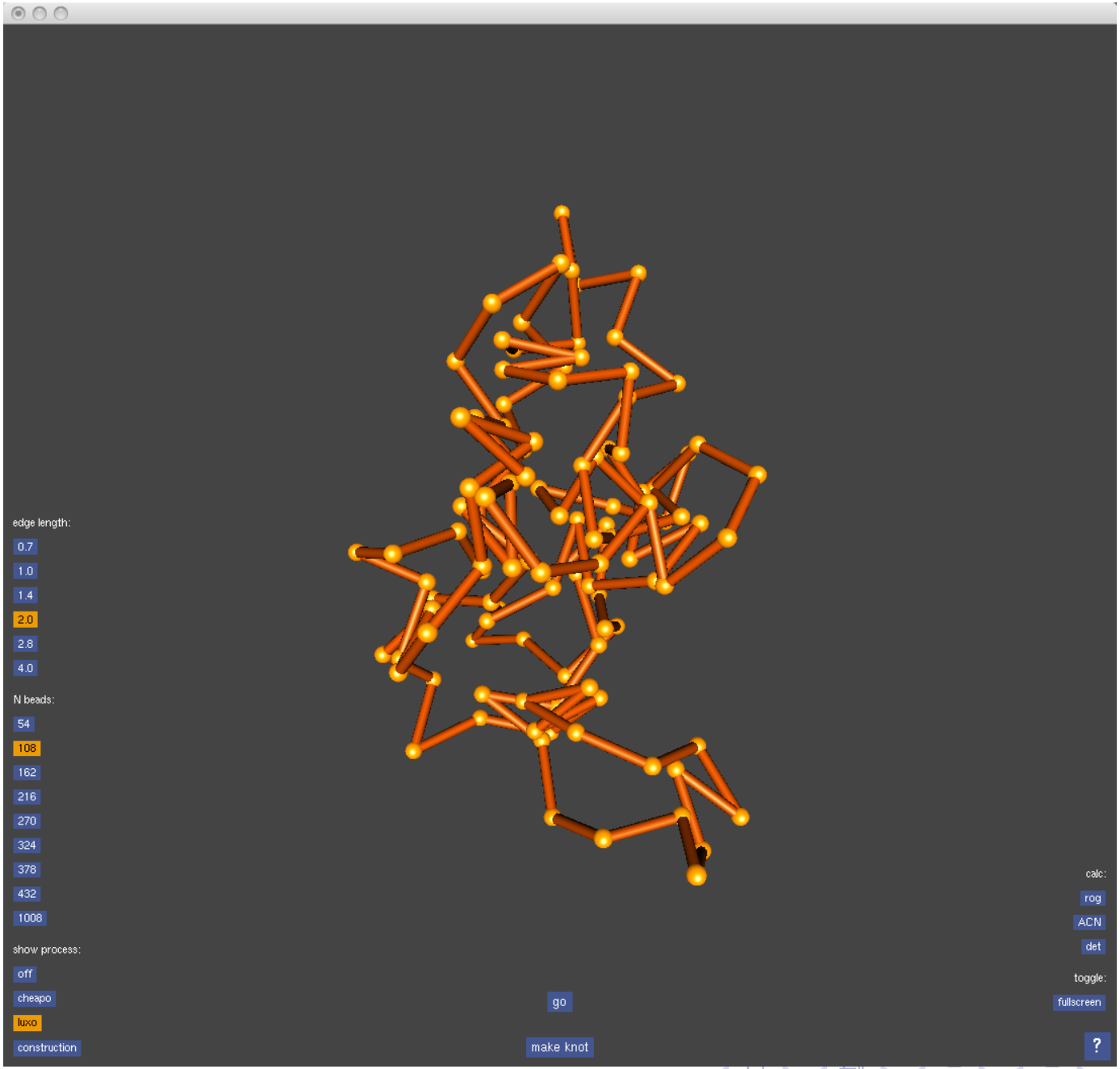


Knotplot can "relax" knots in 3D, smoothing out sharp bends.



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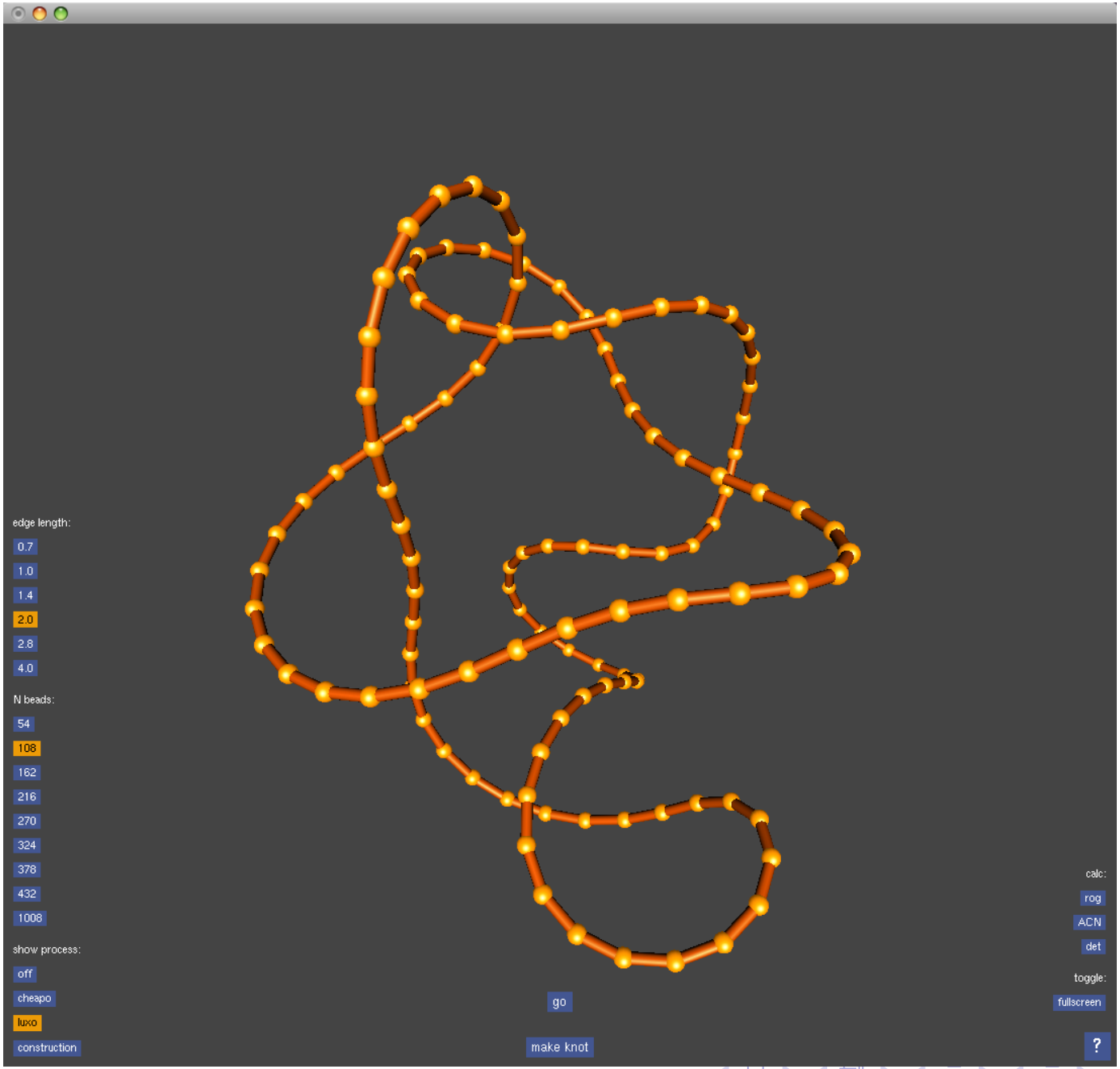

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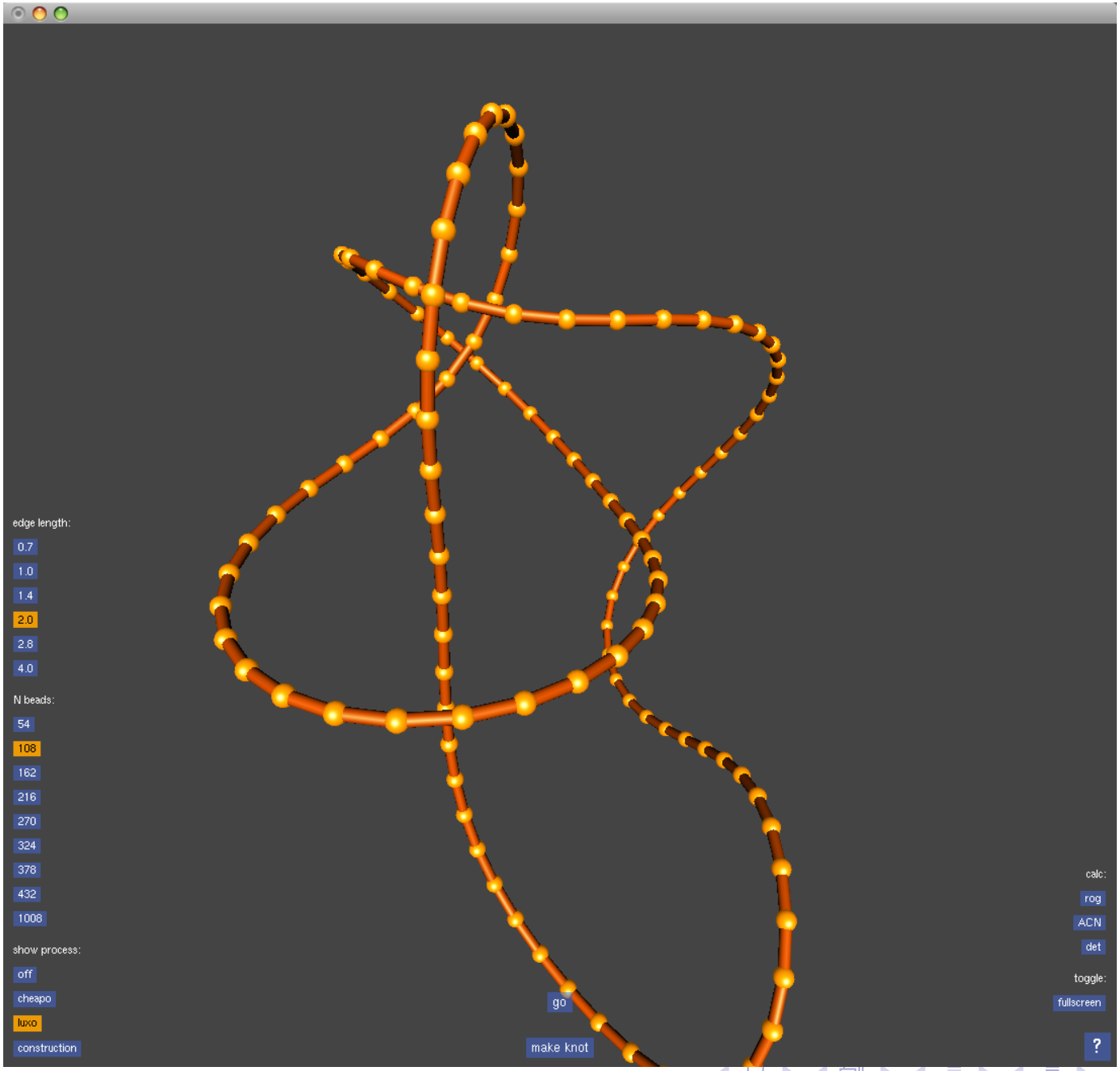

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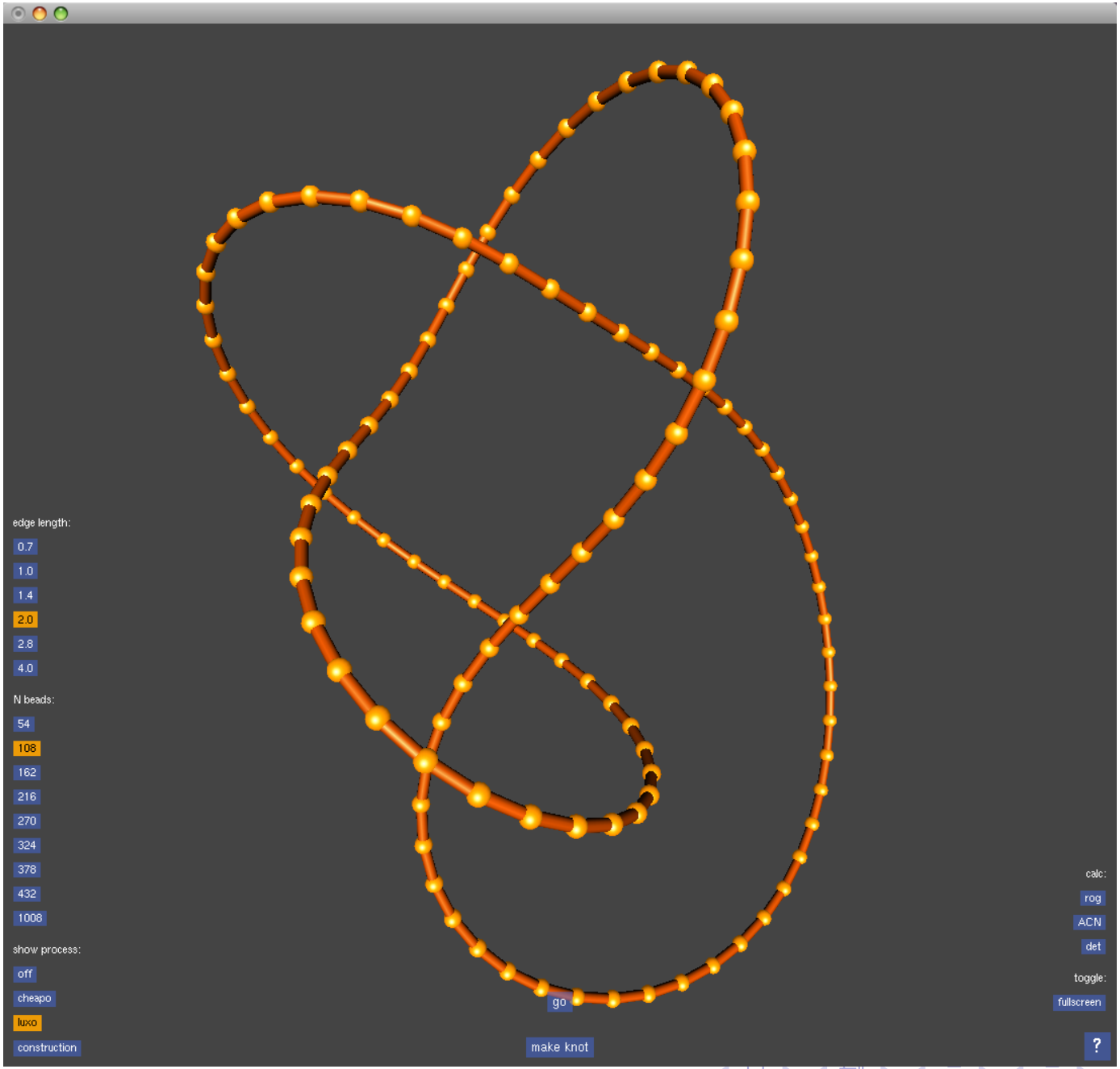
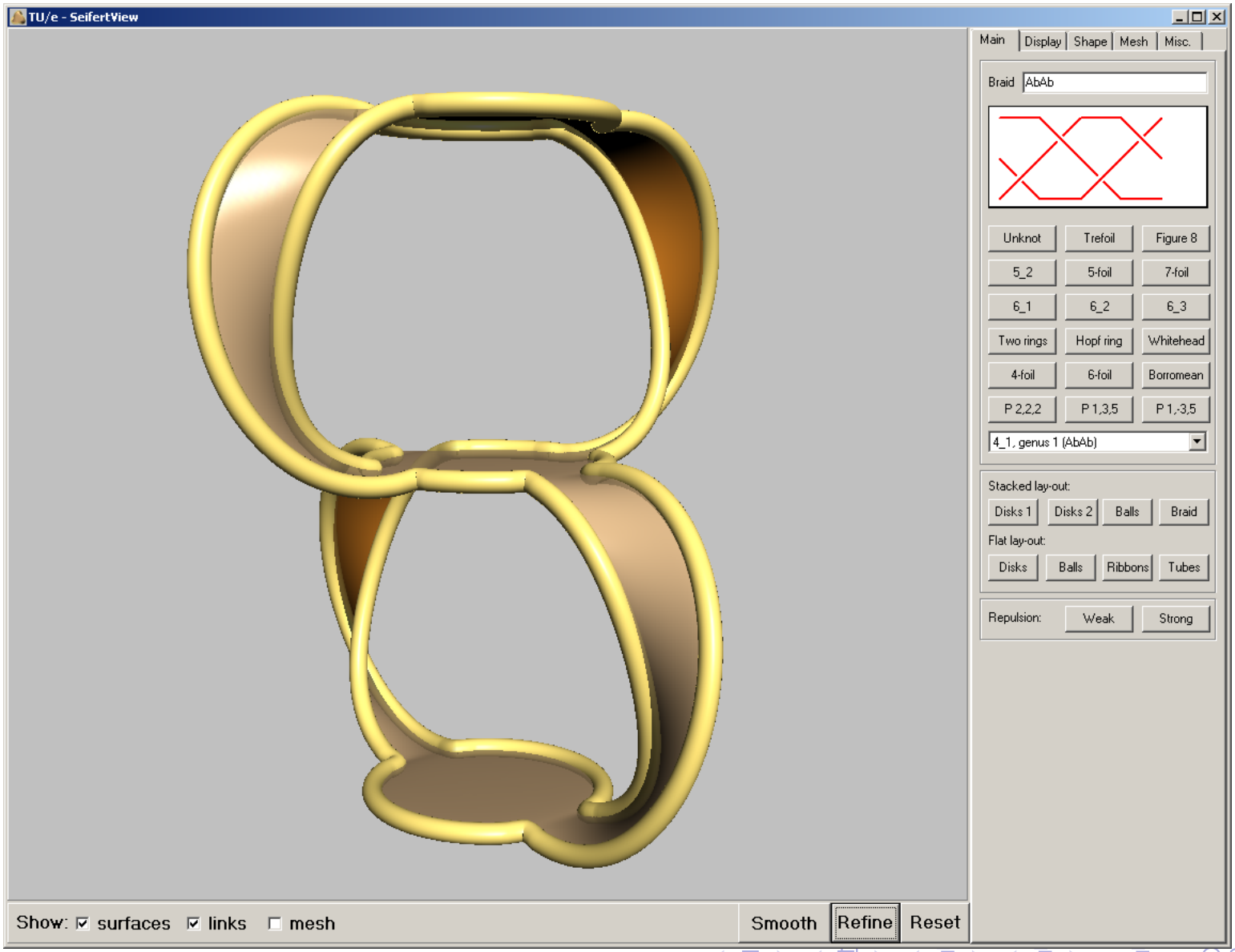


SeifertView

(free, Windows only.)



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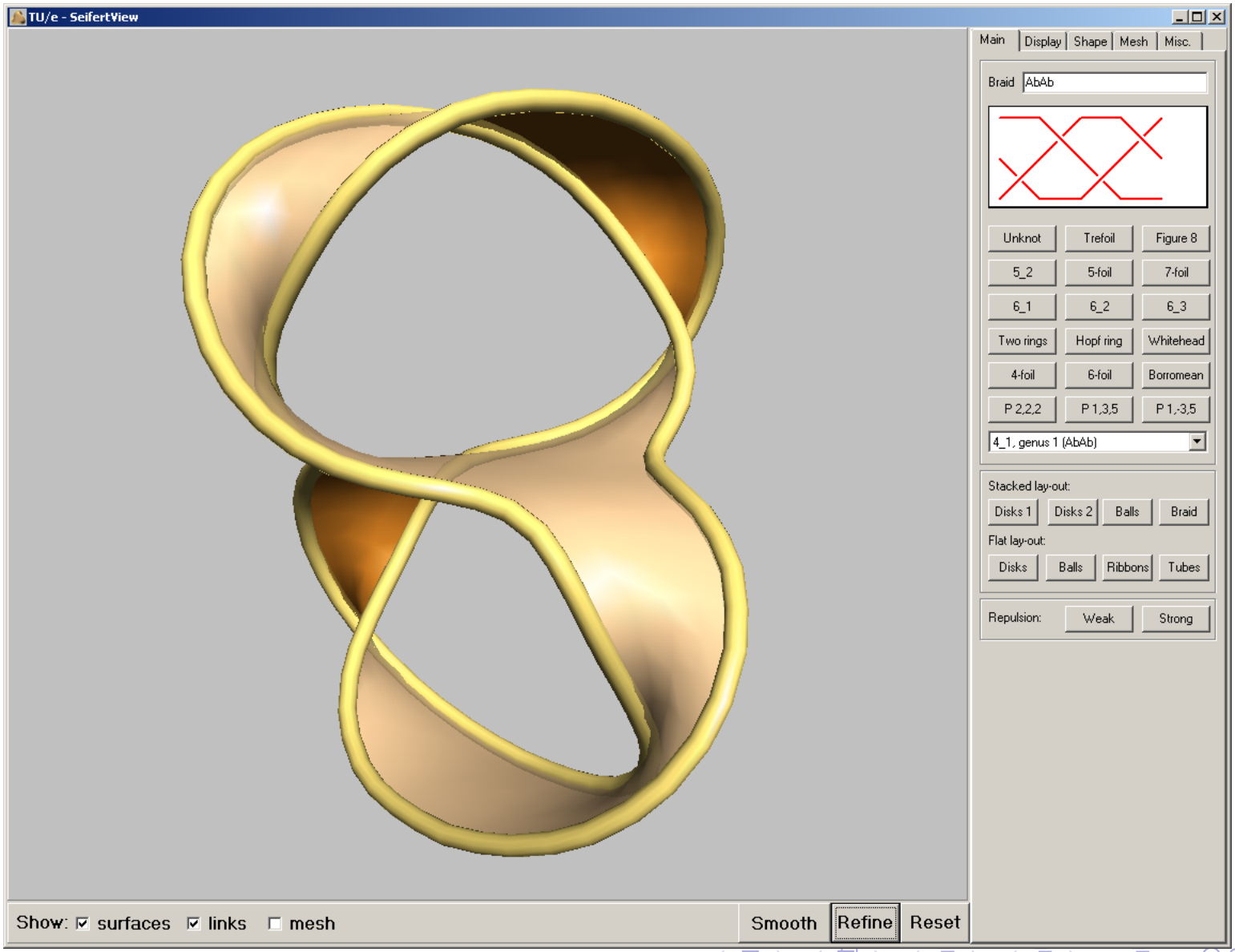

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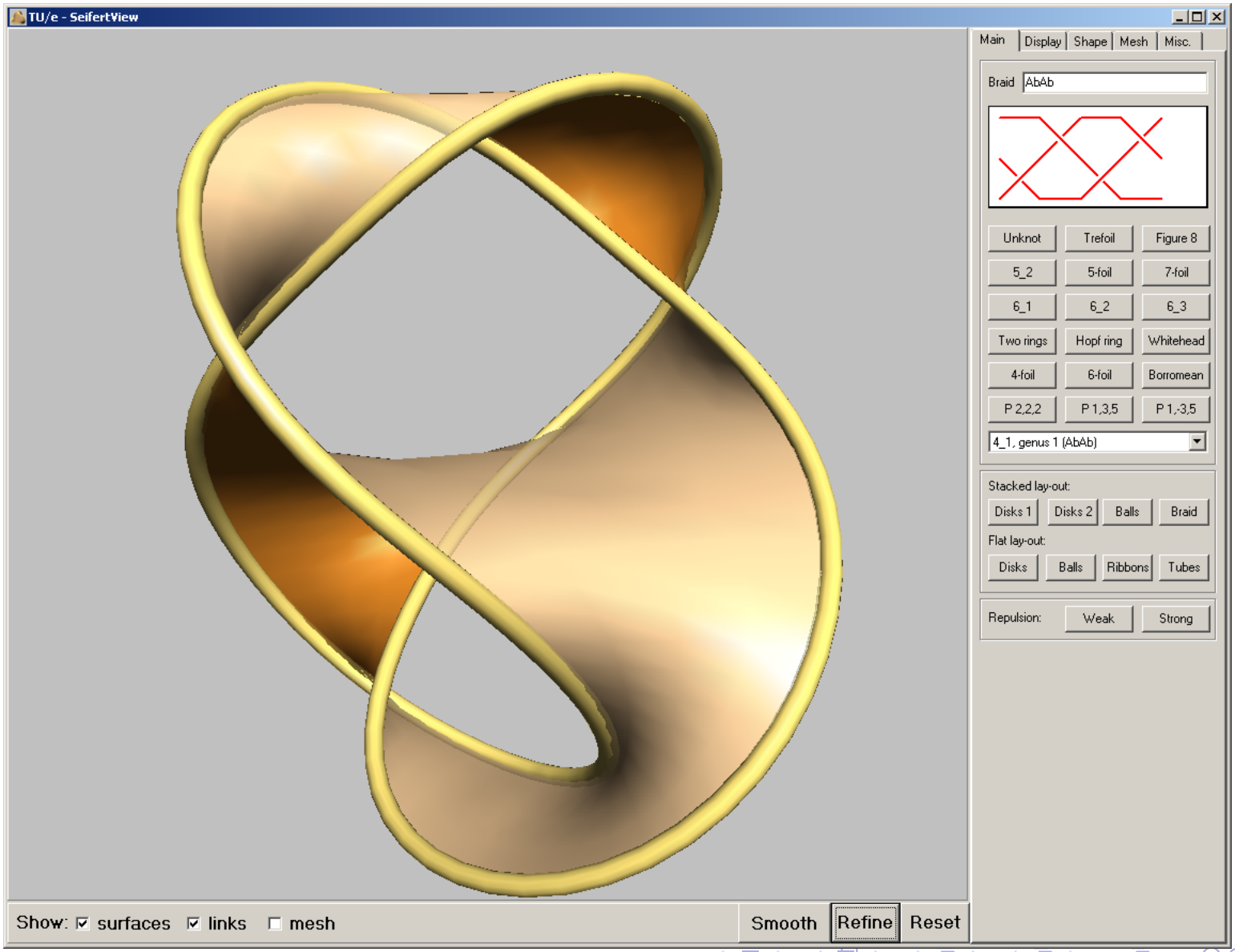
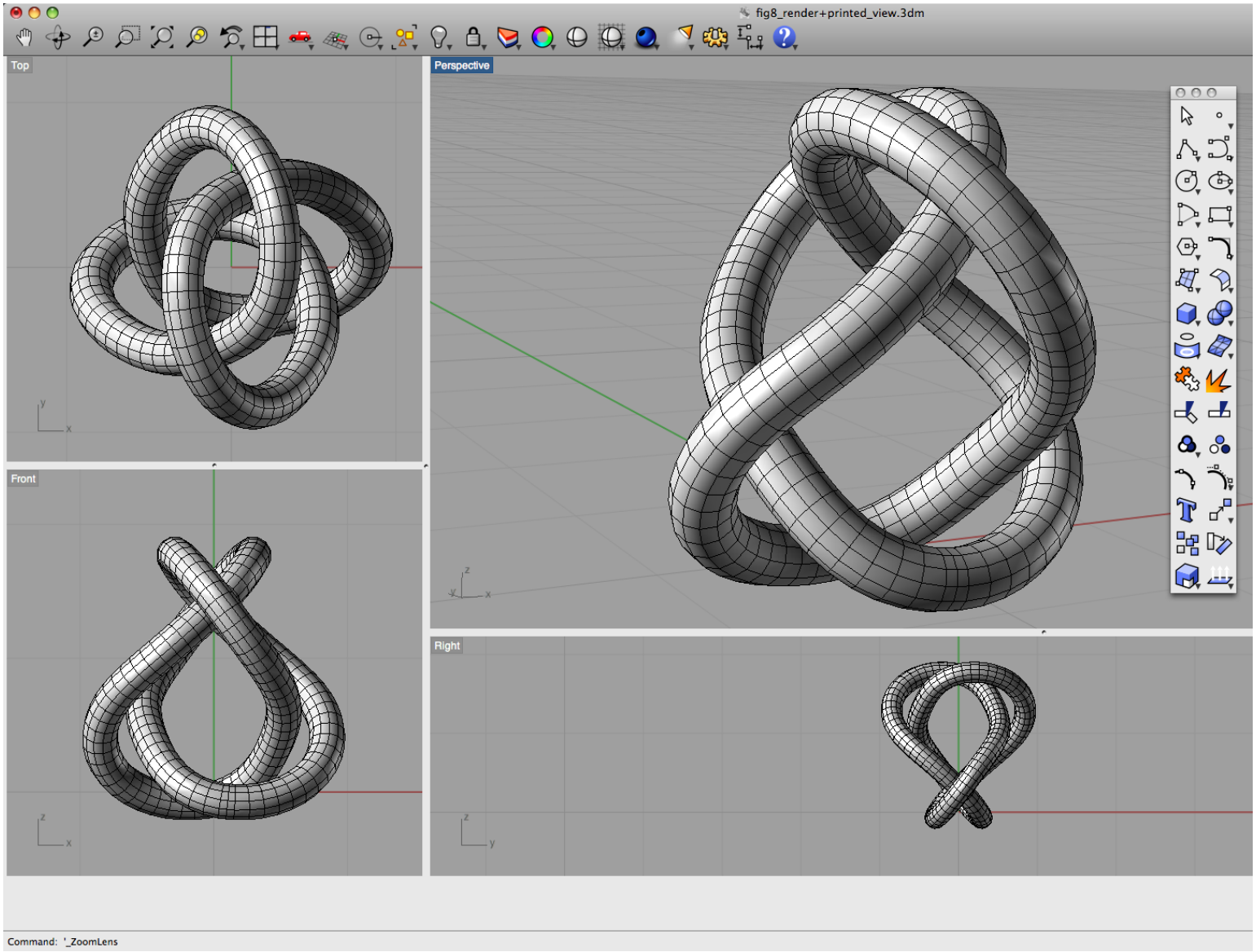


3D printing



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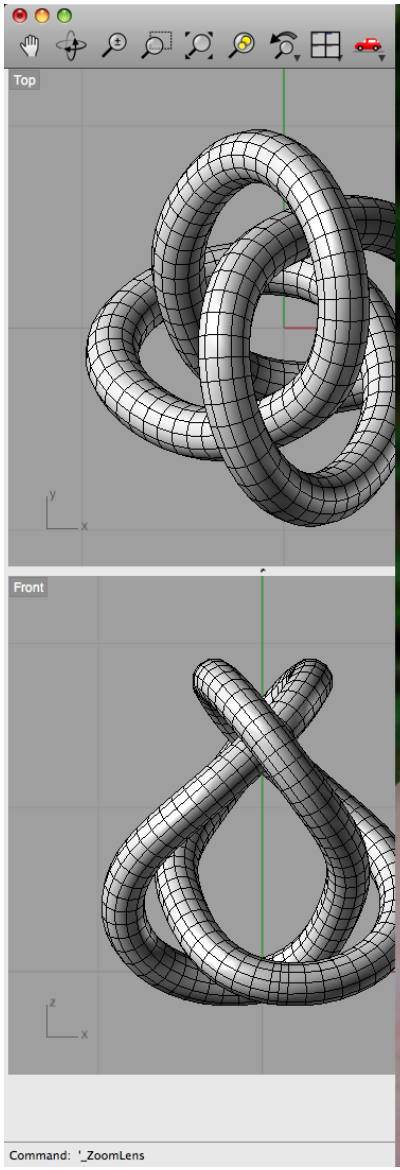
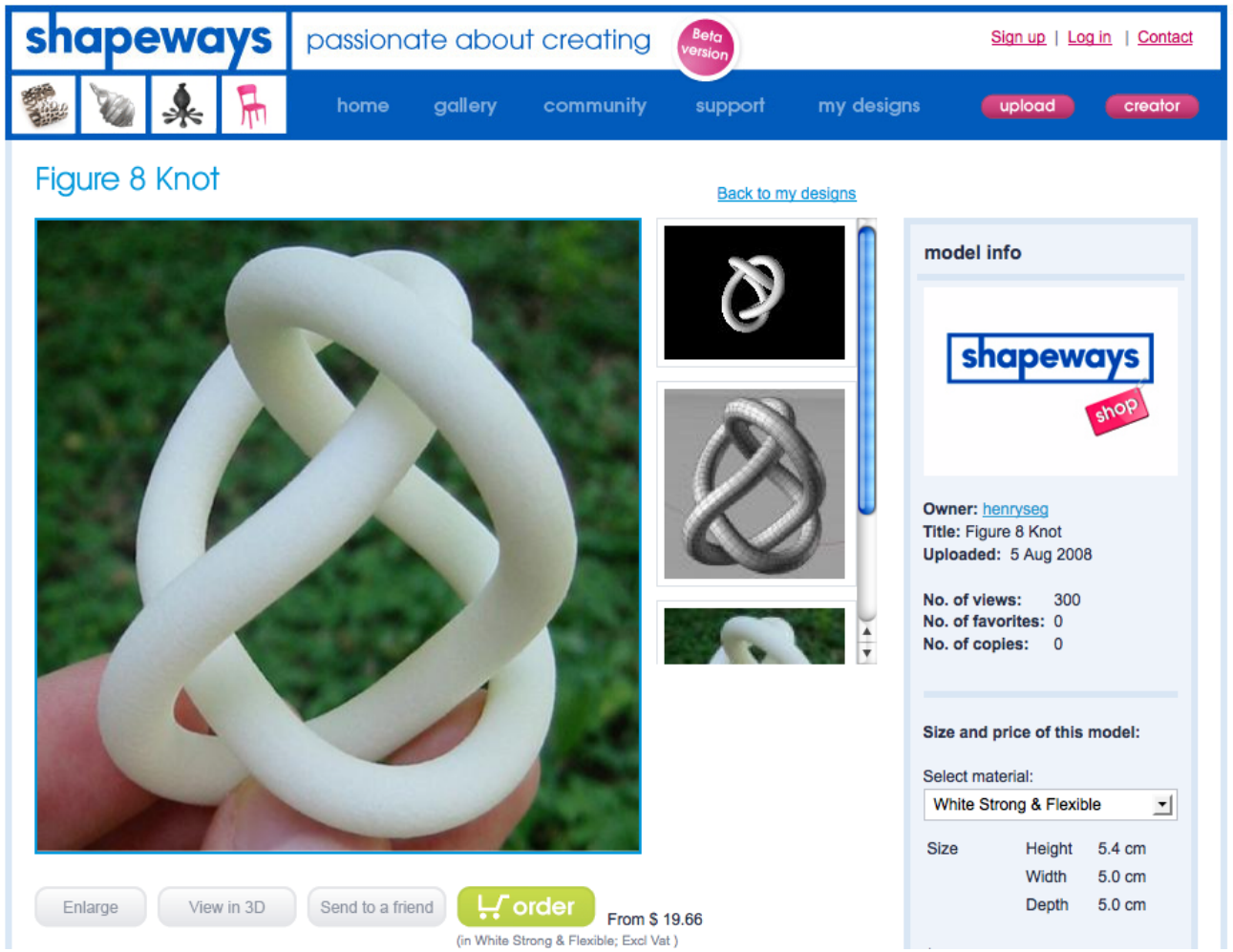
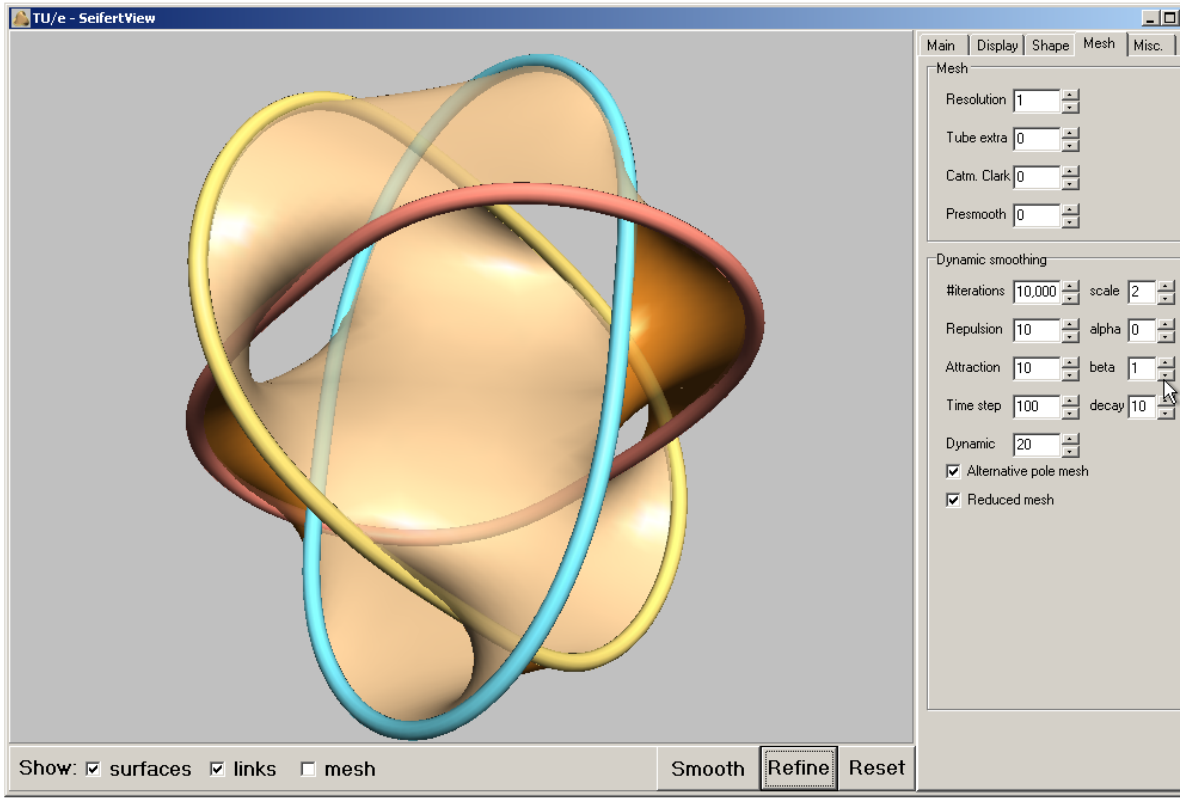


Shapeways.com

Shapeways is a website that lets you upload a 3D model and buy a real life plastic printed copy.



Or, if you are a metal print sculptor such as Bathsheba Grossman, you can make them in bronze.



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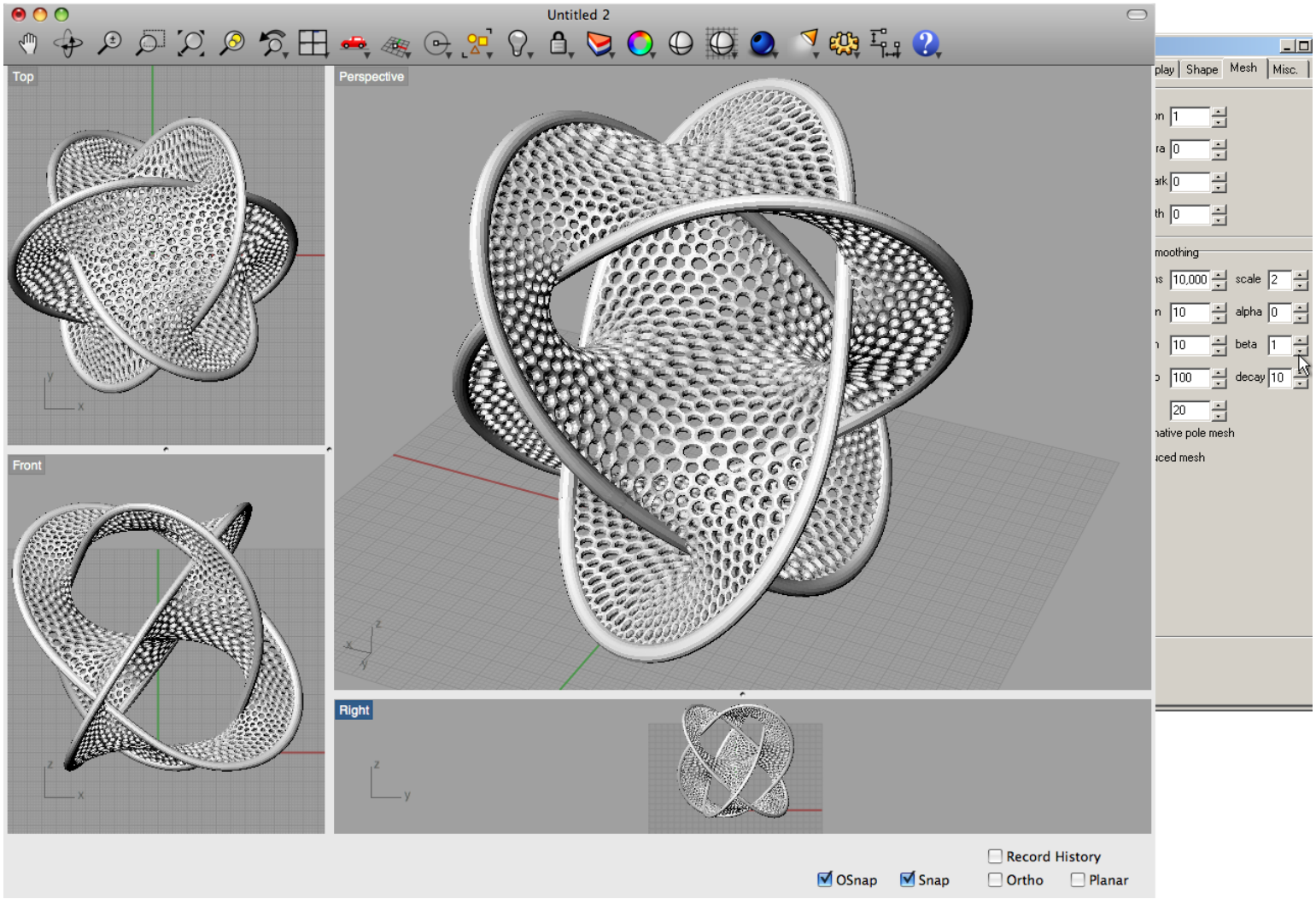

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Bathsheba Sculpture

Borromean Rings

Home
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Sculpture
Mini Sculpture
• Math
Mini Math
Crystal Shop
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Biology
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Click to rotate

This is one of a delightful class of objects known as Seifert surfaces. Every knot and link (in math are closed loops, links are assemblages of knots) has a continuous surface which it is the edge of introduction to these surfaces, along with free software to generate them, are at the SeifertView

These surfaces are often beautiful, especially for symmetrical knots and links, and here I've produ sweeter ones. This surface has three edges, each a simple closed loop, which are locked togethe form called the Borromean Rings. Named after its use in an Italian coat of arms, these three ring together inextricably although no two of them are linked. Their Seifert surface twists through the