 

# Squares that look round: Transforming Spherical Images


Saul Schleimer
Mathematics Institute
University of Warwick

Henry Segerman
Department of Mathematics
Oklahoma State University

# But first... Himmeli

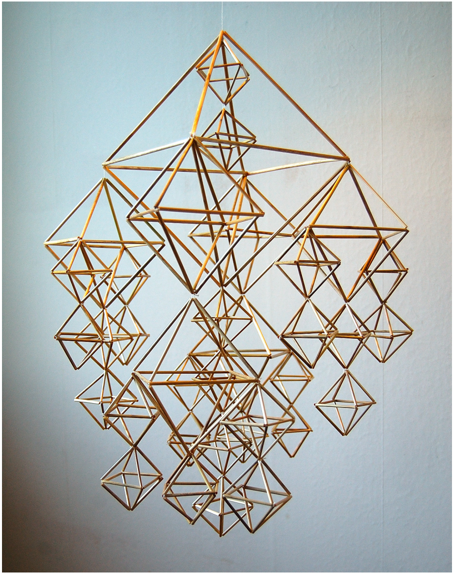

**Photo credit**: `http://kaylovesvintage.blogspot.de`

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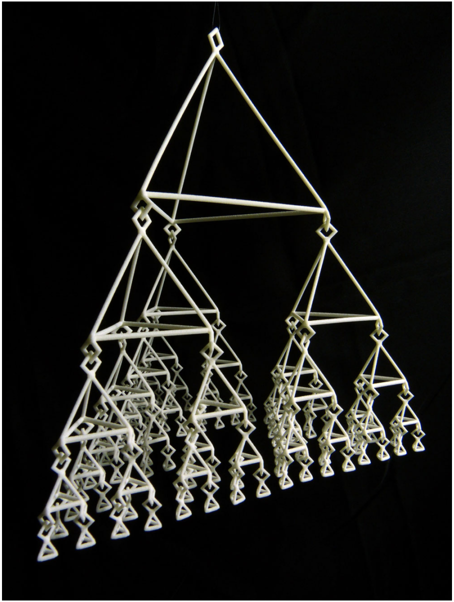

Joint work with Marco Mahler.

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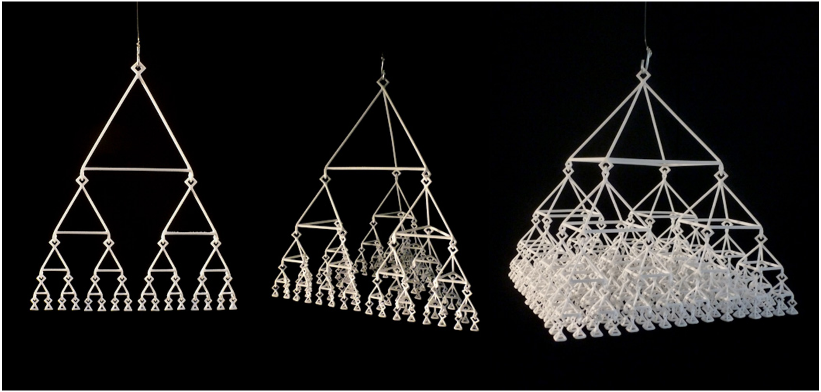

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# Equirectangular projection

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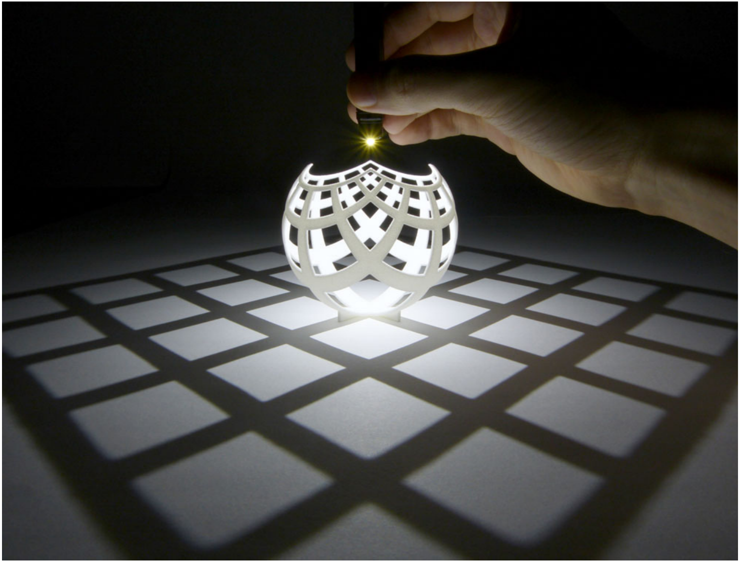
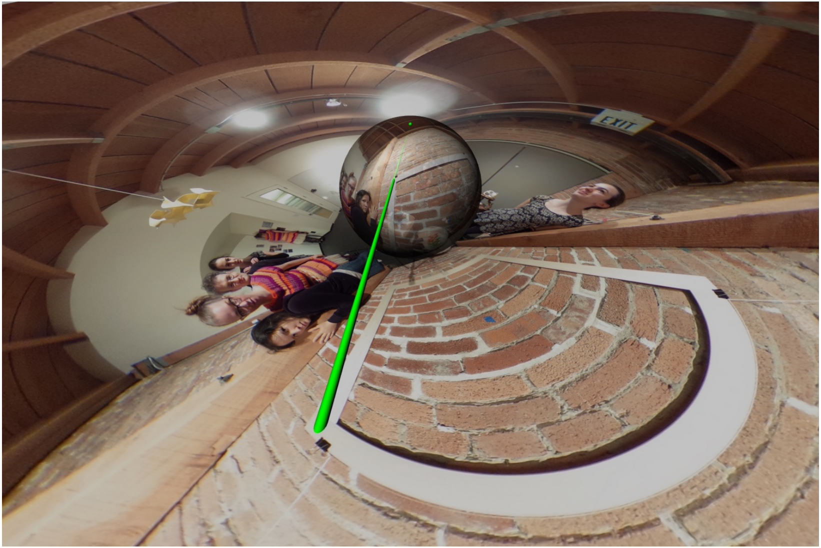
# Stereographic projection $\rho : S^2 \to \widehat{\mathbb{C}}$

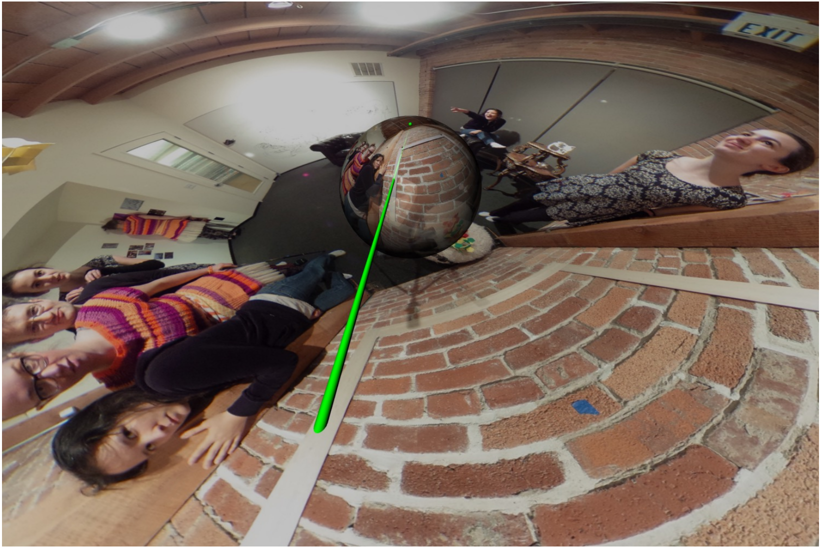


$$\rho(u, v, w) = \frac{u + iv}{1 - w}$$

# Stereographic projection

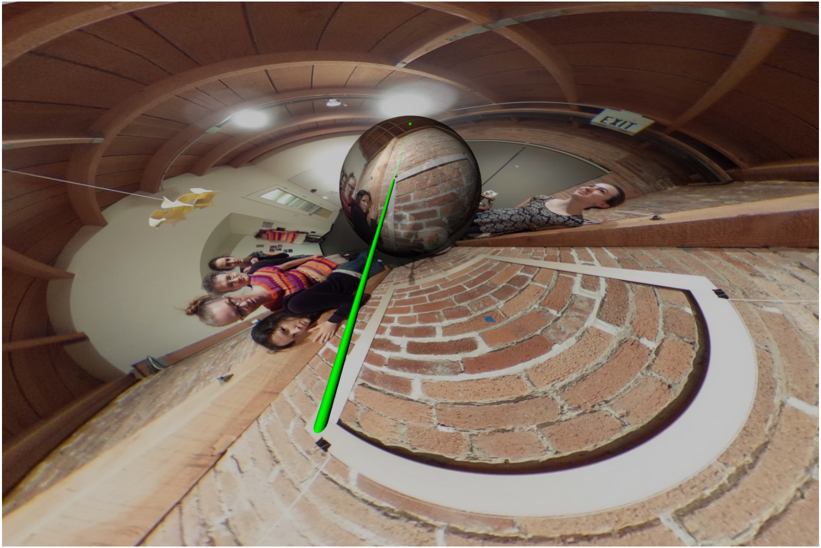


# Transform by $z \mapsto 2z$ (or pull back by $z \mapsto z/2$)




EXIT

# Pull back by $z \mapsto z^2$



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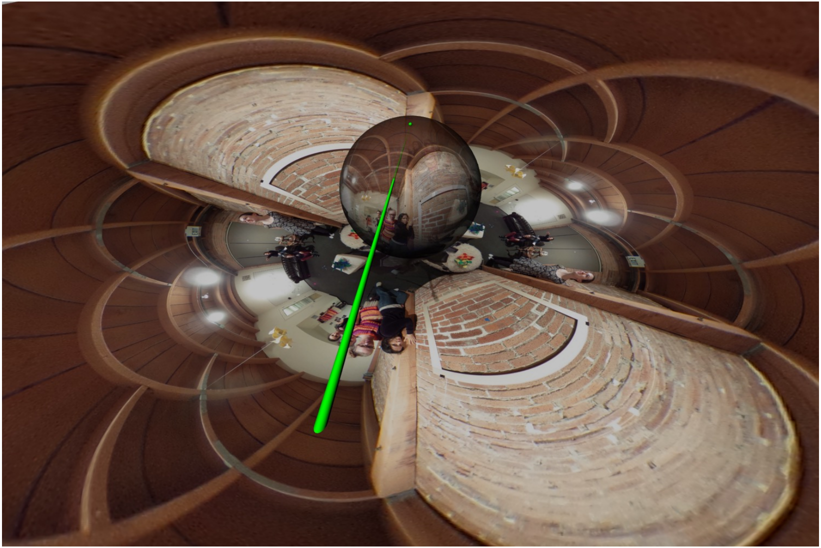


EXIT
EXIT

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# The Droste effect

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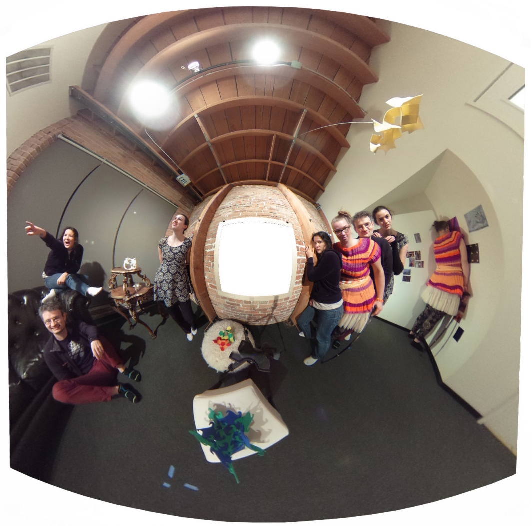
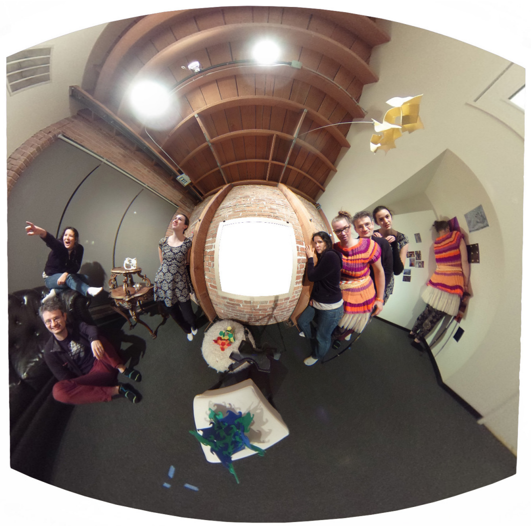
# Droste annulus



# Droste annulus



Apply log, then tile horizontally, apply exp.

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# Twisted Droste effect (Escher, De Smit-Lenstra)

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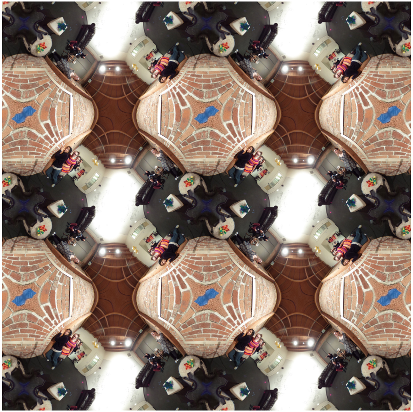
# Other kinds of “twist”, in analogy with the Droste effect

The Weierstrass $\wp$–function (for the square lattice) can be given as

$$\wp_i(z) = \frac{1}{z^2} + \sum{}' \left( \frac{1}{(z-w)^2} - \frac{1}{w^2} \right),$$

where the sum ranges over the non-zero Gaussian integers $w \in \mathbb{Z}[i]$.



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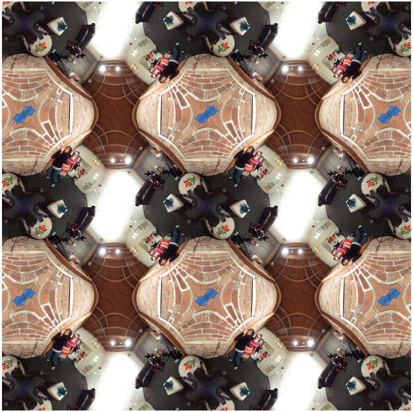
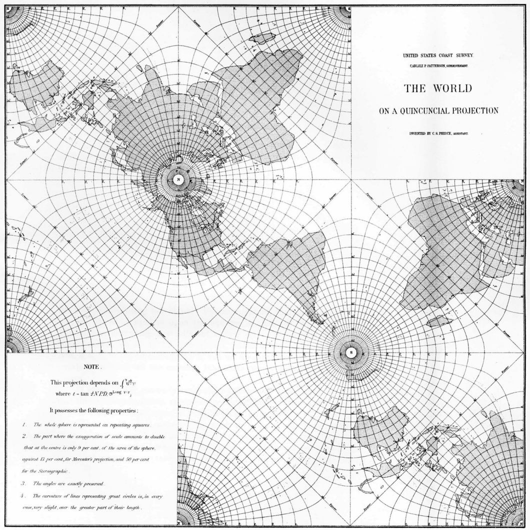


The function is doubly periodic:

$$\wp_i(z+1) = \wp_i(z+i) = \wp_i(z),$$

so we can view it as a map from the torus to $\widehat{\mathbb{C}}$.

# Charles Sanders Pierce used the Weierstrass $\wp$–function on spherical images in 1879.



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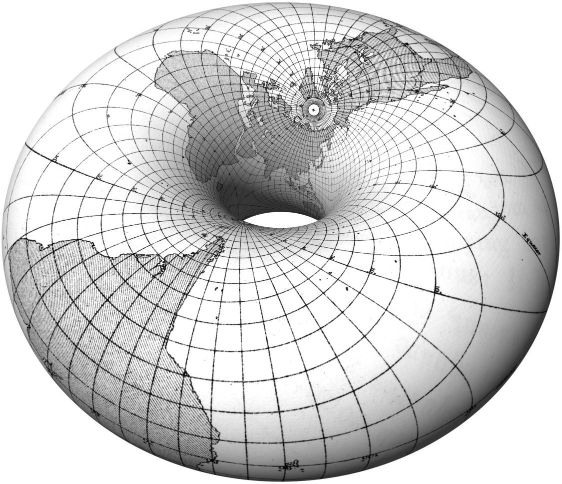
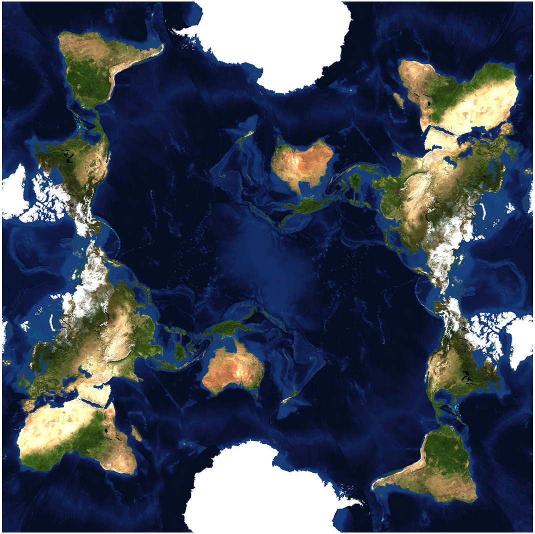


https://skfb.ly/NJRx

# Our version of a torus Earth



# Our version of a torus Earth

https://skfb.ly/MYpC

# Tile, take a different square

Scale by $1 + i$

## Tile, take a different square, then map back to the sphere using a Schwarz-Christoffel map.

Scale by $1 + i$, composition is $z \mapsto \frac{i}{2}(-z + 1/z)$.

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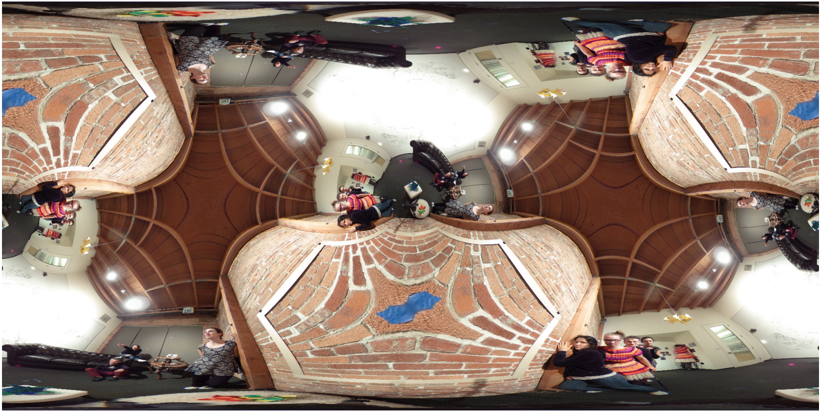

Scale by 2, composition is $z \mapsto \frac{(z^2+1)^2}{4z(z^2-1)}$.

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## Tile, take a different square, then map back to the sphere using a Schwarz-Christoffel map.


Scale by $2+i$, composition is $z \mapsto z\frac{((-1+2i)+z^2)^2}{(-i+(2+i)z^2)^2}$.

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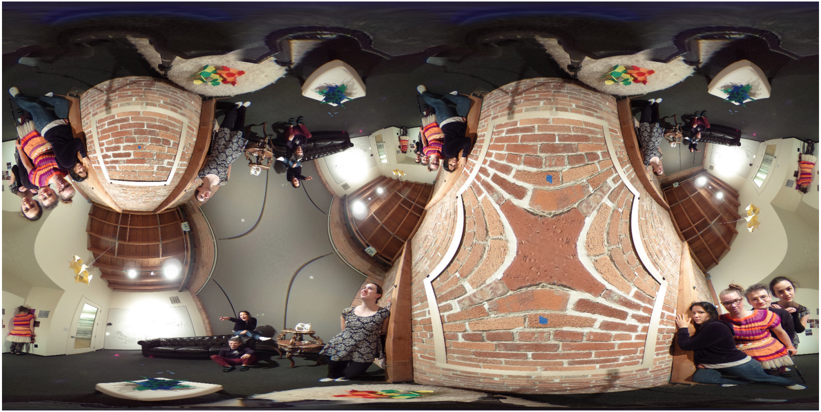
# Hexagonal variation

Instead, we can pull back by the Weierstrass function $\wp_\omega$, where $\omega = e^{\pi i/3}$, giving a hexagonal torus.

# Tile, take a different hexagon

Scale by $1 + \omega$

# Tile, take a different hexagon, then map back to the sphere using a Schwarz-Christoffel map.



Scale by $1+\omega$, composition is $z \mapsto \frac{z^3+\sqrt{2}}{3\omega\cdot z^2}$.

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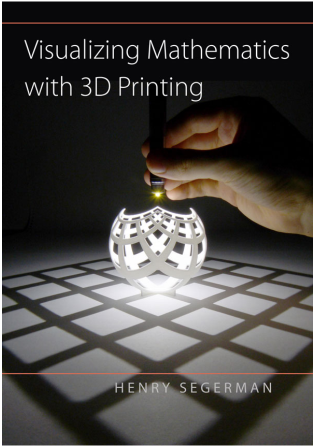
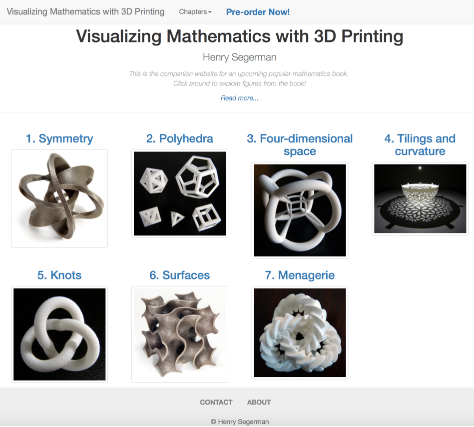
# Book: Visualizing Mathematics with 3D Printing




http://3dprintmath.com

# Thanks!

- Videos at youtube.com/user/henryseg
- Paper at http://archive.bridgesmathart.org/2016/bridges2016-15.html
- (Some) source code at github.com/henryseg/spherical_image_editing