The Sunflower Spiral and the Fibonacci Metric

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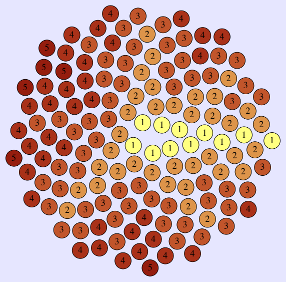
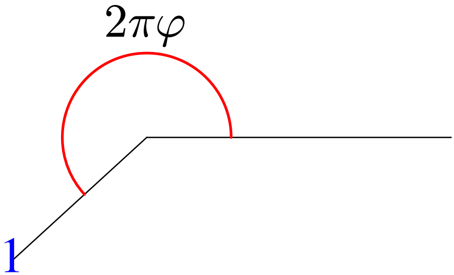


The Sunflower Spiral

Let $S(n) = (r(n), \theta(n)) = (\sqrt{n}, 2\pi\varphi n)$, where $\varphi = \frac{\sqrt{5}-1}{2} = \Phi - 1 \approx 0.618$, and $\Phi = \frac{\sqrt{5}+1}{2}$ is the golden ratio.



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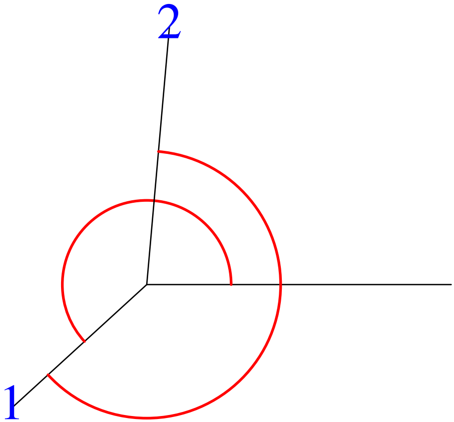

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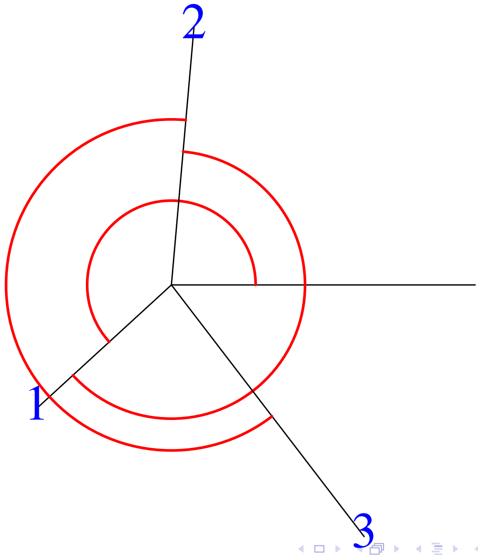

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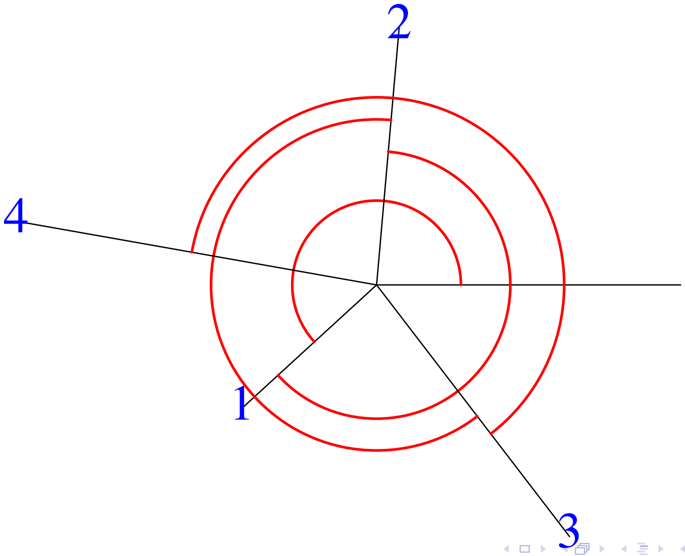

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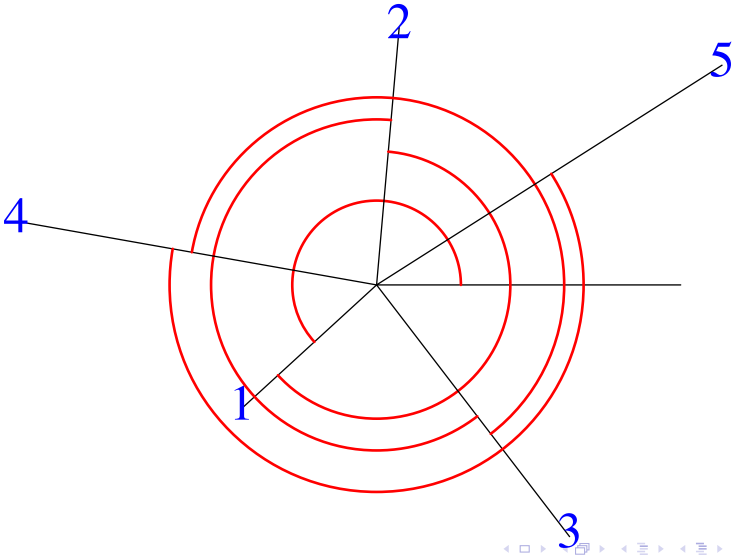

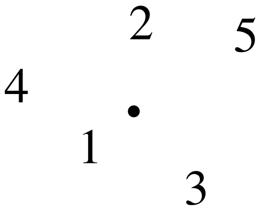
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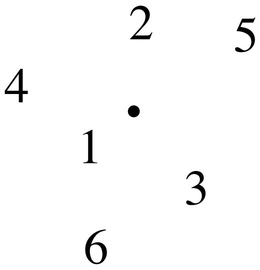
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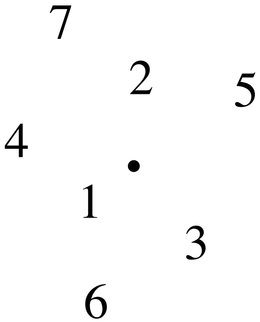
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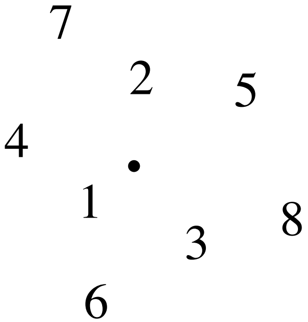
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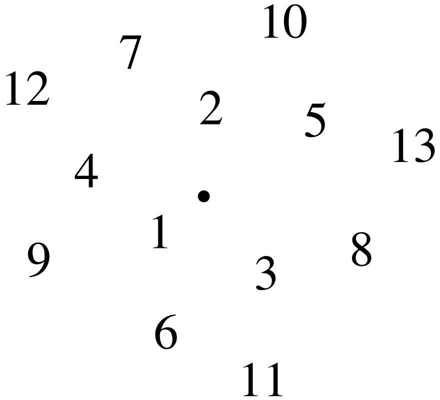
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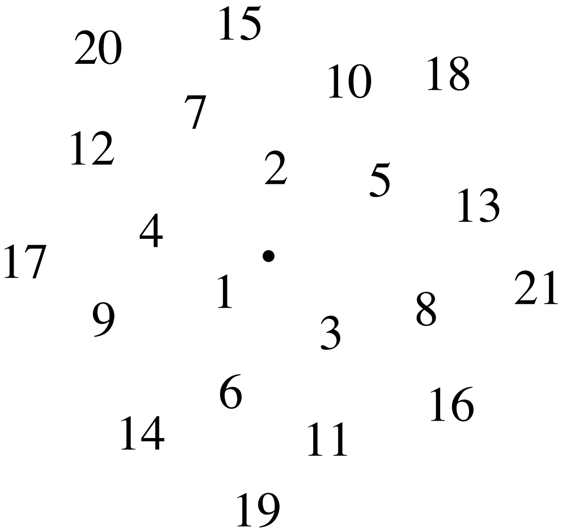
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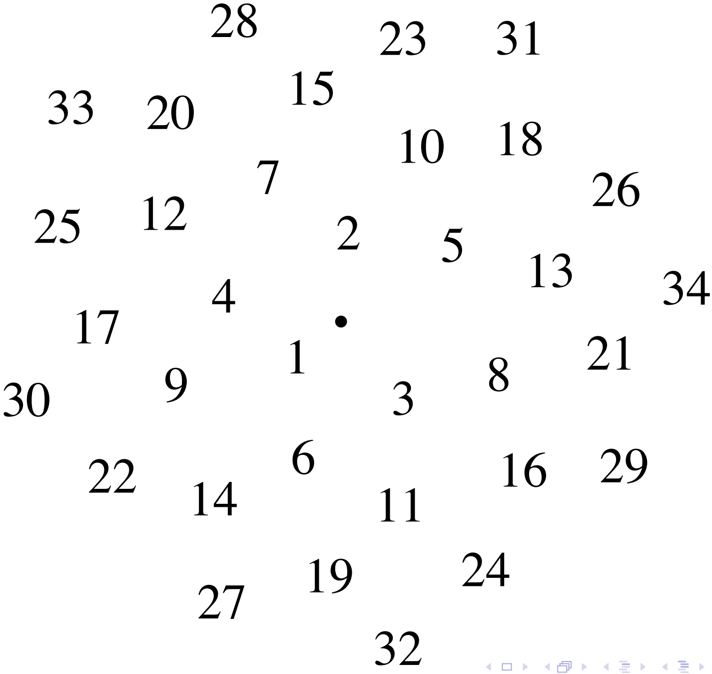
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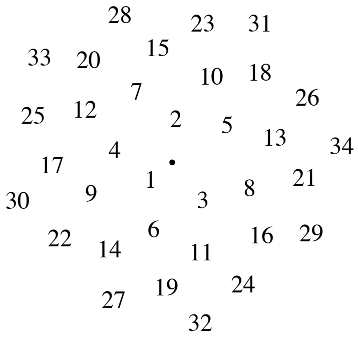
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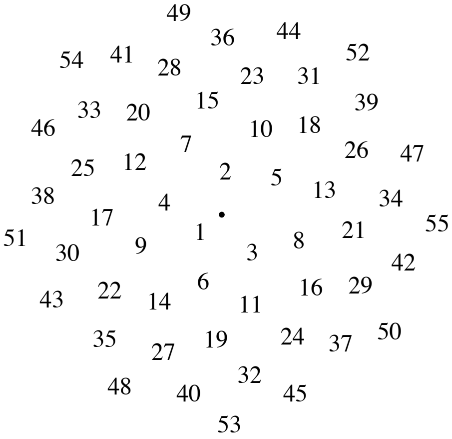
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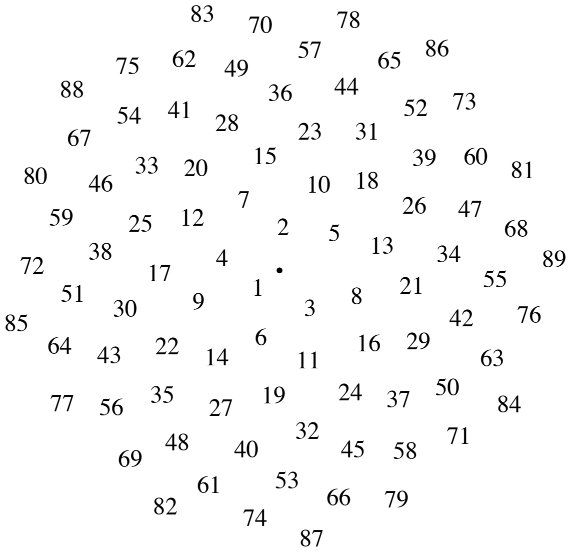

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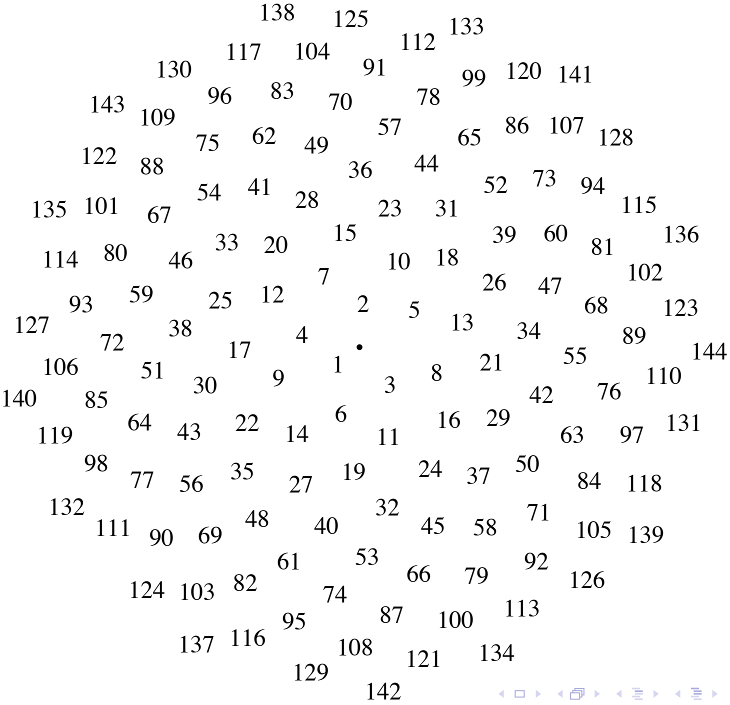
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85 42
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119 43 14 11 16 63 97
98 24 50
77 56 35 19 37 84 118
27
132 32 71
111 48 40 45 58 105 139
90 69
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This sequence of points models many patterns in nature, in particular the florets on a sunflower head.

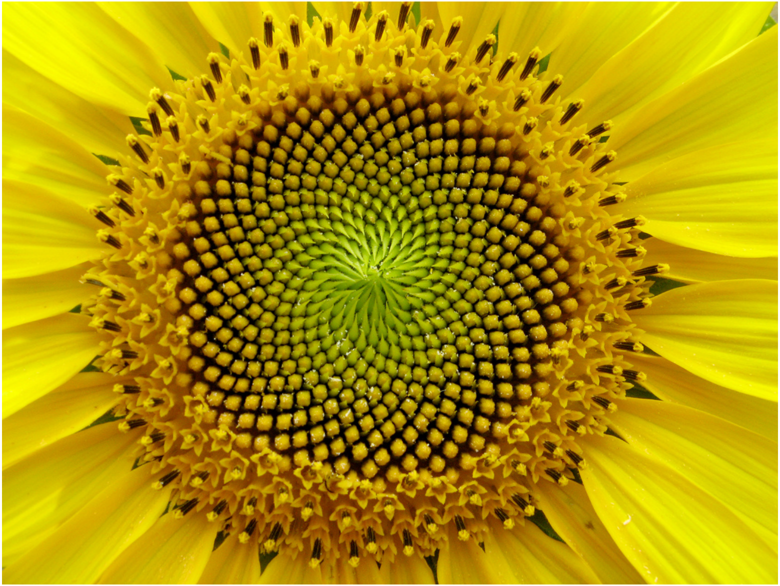
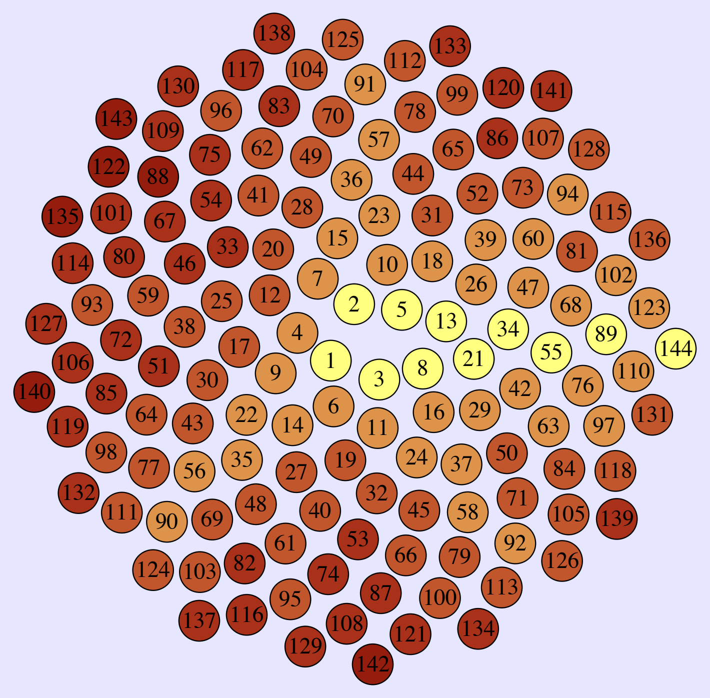


Photo credit: http://www.flickr.com/photos/lucapost/694780262/

Fibonacci Metric

Let $M : \mathbb{N} \rightarrow \mathbb{N}$ be a function, $M(n)$ is the minimal number of Fibonacci numbers F_i needed in order to sum to n.

$$\begin{aligned}
1 &= 1 & M(1) &= 1 \\
2 &= 2 & M(2) &= 1 \\
3 &= 3 & M(3) &= 1 \\
4 &= 3+1 & M(4) &= 2 \\
5 &= 5 & M(5) &= 1 \\
6 &= 5+1 & M(6) &= 2 \\
7 &= 5+2 & M(7) &= 2 \\
8 &= 8 & M(8) &= 1 \\
9 &= 8+1 & M(9) &= 2 \\
10 &= 8+2 & M(10) &= 2 \\
11 &= 8+3 & M(11) &= 2 \\
12 &= 8+3+1 & M(12) &= 3
\end{aligned}$$



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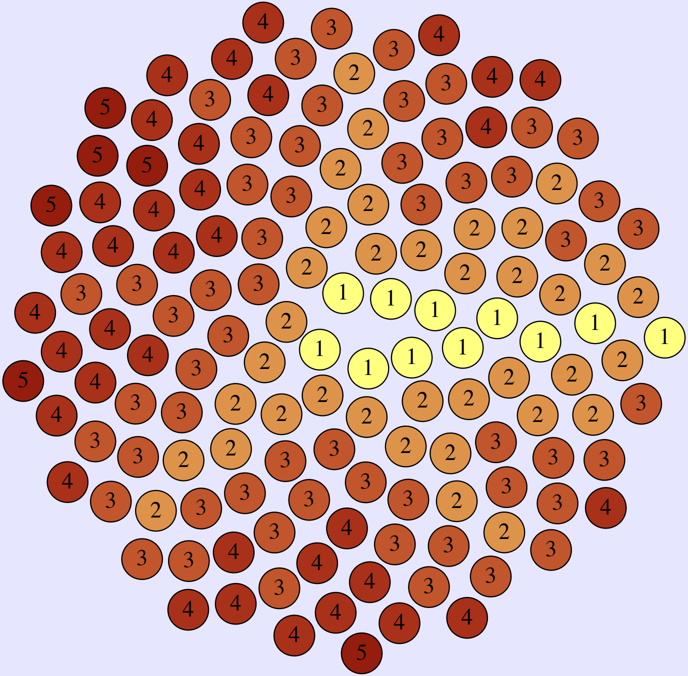
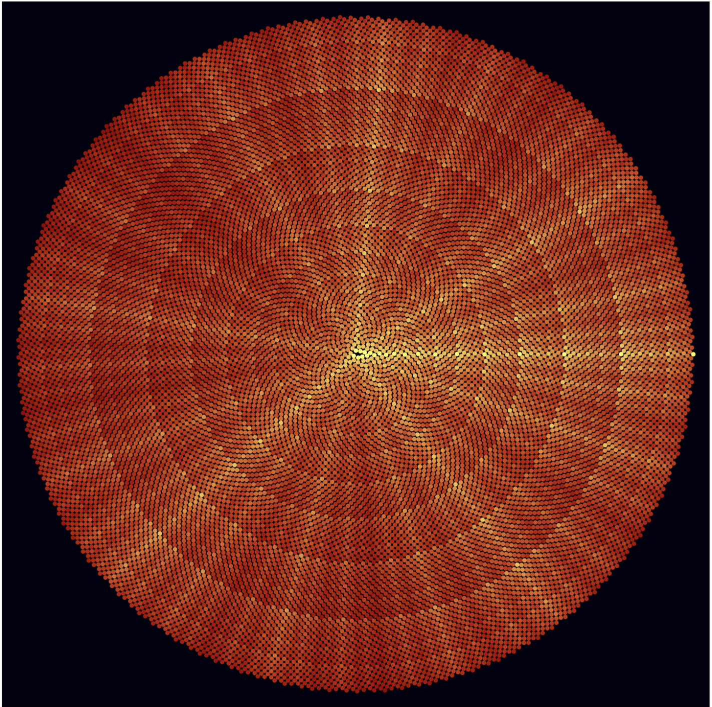
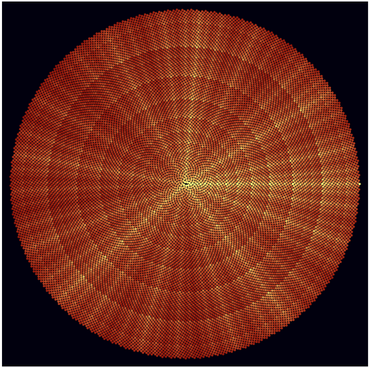




Where do the patterns come from?

1. Radial spokes
2. Circular tree rings



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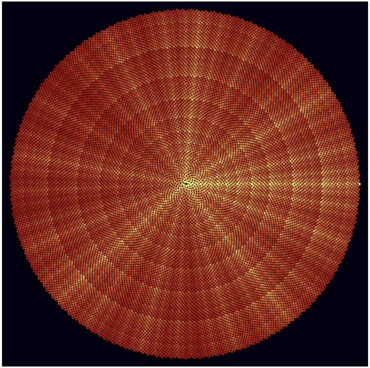
For the spoke at $\theta = 0$:

$$
\begin{aligned}
F_k &= \frac{\Phi^k - (1-\Phi)^k}{\sqrt{5}} \\
&\Downarrow \\
F_k - \varphi F_{k+1} &= (-\varphi)^{k+1} \\
&\Downarrow \\
\theta(F_{k+1}) &= 2\pi\varphi F_{k+1} \\
&= 2\pi\left(F_k - (-\varphi)^{k+1}\right)
\end{aligned}
$$



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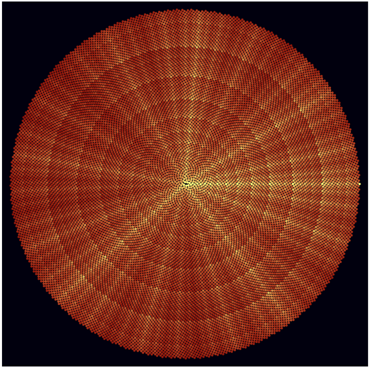

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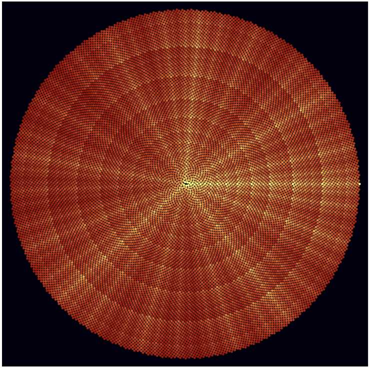
\end{aligned}
$$

$(-\varphi)^{k+1}$ is small, and so for large k, $\theta(F_{k+1})$ is almost a multiple of 2π.

Thus the Fibonacci numbers themselves are near $\theta = 0$.

Where do the patterns come from?

Sums of Fibonacci numbers have angles the sum of the angles of the Fibonacci numbers, so sums of a small number of large Fibonacci numbers are also near $\theta = 0$. This makes up other points of the $\theta = 0$ spoke.



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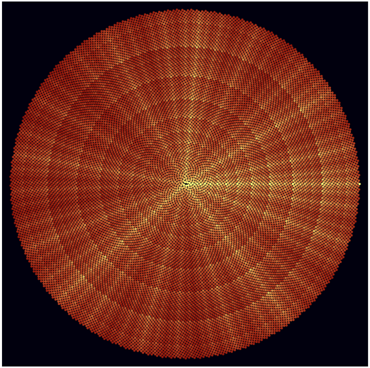
Other spokes are rotations of the $\theta = 0$ spoke, formed by adding a small number to all of the numbers in the $\theta = 0$ spoke.



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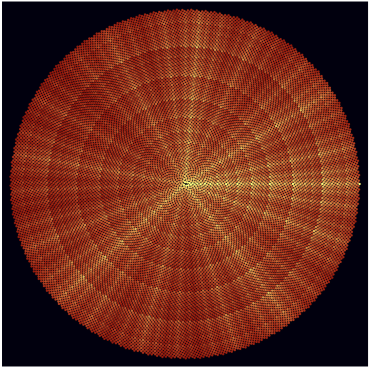
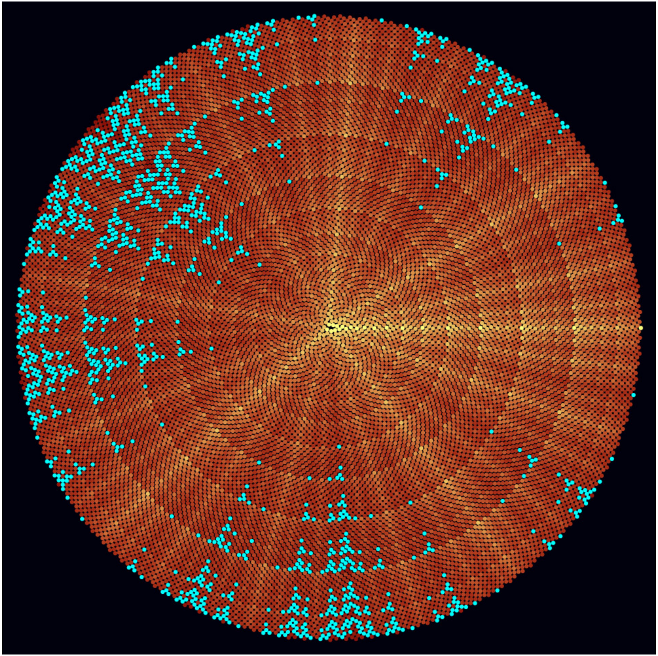


For the tree rings: Just after a large number m with small $M(m)$, there will be many numbers n for which the minimal $M(n)$ is achieved using m and some small number of additional Fibonacci numbers, because $n - m$ is small.



Thanks!